The Marseille Software

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TMR
Training and Mobility of Researchers

Overview

- Non adaptive framework:
  $\rightsquigarrow$ Filters
  - orthogonal, biorthogonal, Harten construction, wavelets and operators, wavelets on the interval
  - tree algorithm in a periodic case

  $\rightsquigarrow$ Redundant B-Spline analysis

  $\rightsquigarrow$ Preconditioning

- Adaptive / Non linear framework:
  $\rightsquigarrow$ Adaptive tree algorithm in 1D and 2D

  $\rightsquigarrow$ Evaluation of non linear functional

  $\rightsquigarrow$ Adaptative numerical resolution of PDE's

General Information:

http://taoume.irphe.univ-mrs.fr/mnm

Filter for Orthogonal Wavelets

1. Module

Filter

computation of filter coefficients

2. Author

Jürgen Vorloeper

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3. Description

- orthogonal spline wavelets on $[0,1]$, periodic b.c. Basdevant/Perrier '89
- m oder of splines, m pair
  $\rightsquigarrow$ m vanishing moments, regularity C^{m-2}
- symmetry; global support, however exponential decrease
  $\rightsquigarrow$ truncate filter $\rightsquigarrow$ numerically compact support
  $\rightsquigarrow$ compute coefficients $(h_r^{(j)})_{r=0,\dots,N}$, $(g_r^{(j)})_{r=0,\dots,N}$, $N < 2^{j-1}$, with

$$\varphi_{j,k} \doteq \sum_{r=-N}^{N} h_r^{(j)} \varphi_{j+1,2k+r}, \quad \psi_{j,k} \doteq \sum_{r=-N}^{N} g_r^{(j)} \varphi_{j+1,2k+r}$$

- filter $(\ell_r^{(j)})_{r=0,\dots,N}$ for interpolation operator

4. Language

C++

5. Input Parameters

m order of splines, N length of filter, j level

6. Output Parameters

- filters $\mathbf{h}^{(j)}$, $\mathbf{g}^{(j)}$, $\mathbf{l}^{(j)}$ and their Fourier transform
- storage: class Mask (cf. IGPMlib)

7. Remark

- routines for FFT from "Numerical Recipes in C"
- includes: class Polynomial, class Mask

8. References

[1] C. Basdevant and V. Perrier. *La décomposition en ondelettes périodiques, un outil pour l'analyse de champs inhomogènes. Théorie et algorithmes.* La Recherche Aérospatiale, Nr. 3, pp. 53–67, 1989.

[2] W. H. Press, S. A. Teukolsky, W. T. Vetterling and B. P. Flannery. *Numerical Recipes in C: The Art on Scientific Computing.* Cambridge University Press, 1st, 2nd ed., 1989, 1993.

Biorthogonal B-Splines filters

1. Module

   splines.f, biortho.f, inter-splines.f

   filters associated to the B-Splines biorthogonal multiresolution analysis $(V_j, \tilde{V}_j)_j$, periodisation in $V_p, \tilde{V}_p$.

2. Author

   Anne–Sophie Piquemal

   email: piquemal@marius.univ-mrs.fr

3. Description

   splines.f: sets $(h_k)_k$, $(g_k)_k$ associated to the scale relations defined by the B-Splines of order m N_m and the associated compact support wavelet ψ.

   biortho.f: sets $(\tilde{h}_k)_k$, $(\tilde{g}_k)_k$ associated to the scale relations defined by the biorthogonal scale function $\tilde{\phi}$, and the associated biorthogonal wavelet $\tilde{\psi}$. Computation only for $m = 4$, due to the truncation of the infinite sets.

   inter-splines.f: interpolation filter $(L_k)_k$ in V_p, associated to N_m.

In the fourier space $\hat{L}_\omega = e^{-i\frac{\omega}{2}} P_{\frac{m}{2}-1}(\sin^2 \frac{\omega}{2})$

where poylnom P_n of degree n is defined by $\frac{P_{n-1}(\sin^2 \omega)}{(\sin\ \omega)^{2n}} = \sum_{l \in \mathbb{Z}} \frac{1}{(\omega + l\pi)^{2n}}$

4. Language

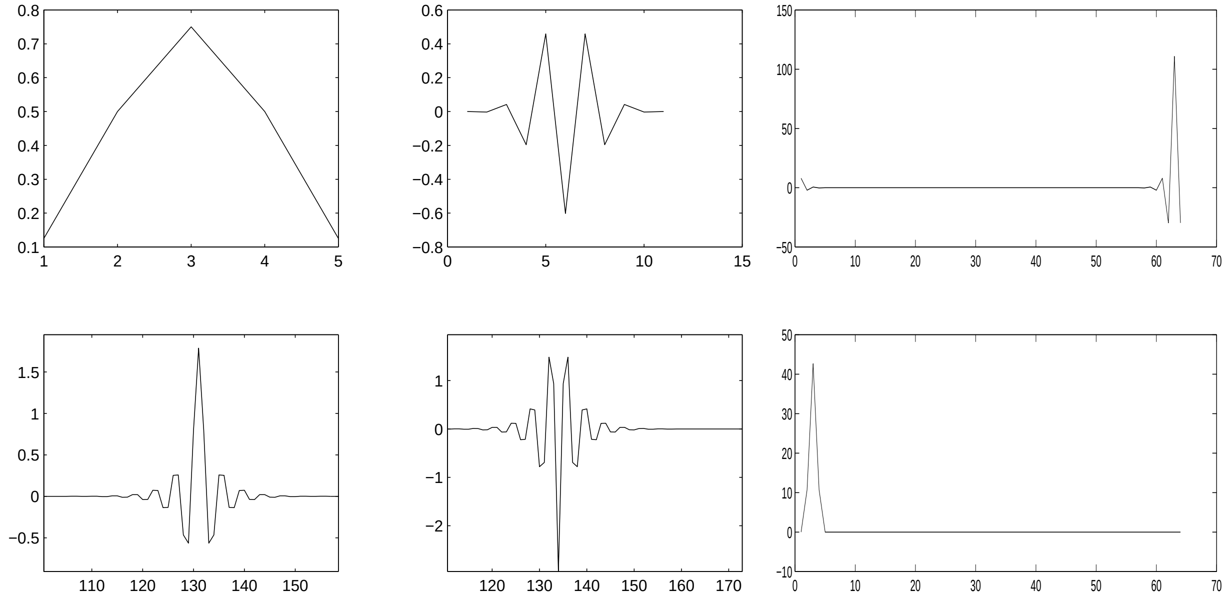
fortran77

5. Input parameters

m order of B-Splines, p dimension of the approximation space (2^p).

6. Output parameters

m=4,p=6



B-Splines biorthogonal MRA and laplacian

1. Module

amrl-1.f, inter-amrl-1.f

filters provided by the action of the laplacian $\triangle = -\frac{\partial^2}{\partial x^2}$ on the B-Splines biorthogonal MRA.

2. Author

Anne–Sophie Piquemal

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3. Description

amrl-1.f: sets $(\tilde{H}_k)_k$ and $(\tilde{G}_k)_k$ associated to the MRA $(\tilde{U}_j)_j$ providing the function $\tilde{\theta} = \Delta^{-1}\psi$

- symbol of Δ: $\sigma(\omega) = -\omega^2$
- function S: $S(\omega) = (1 - e^{i\omega})^2$ equivalent to σ around $\omega = 0$
- symbols associated to the MRA $(\tilde{U}_j)_j$ $\tilde{p}_0$ which defines $(\tilde{H}_k)_k$

$$\tilde{p}_0(\omega) = \frac{1}{4}\frac{S(2\omega)}{S(\omega)} m_0(\omega)$$

and $\tilde{p}_1$ which defines $(\tilde{G}_k)_k$

$$\tilde{p}_1(\omega) = \frac{1}{4}\frac{m_1(\omega)}{S(\omega)}$$

inter-amrl-1.f: interpolation filter $(\tilde{L}_k)_k$ in $\tilde{U}_p$ associated to $\tilde{\tau}$

$$\hat{\tilde{\tau}}(\omega) = \frac{S(\omega)}{\sigma(\omega)}\hat{N}_m(\omega), \quad \hat{\tilde{L}}_\omega = e^{-i\omega(\frac{m}{2}-1)} P_{\frac{m}{2}}(\sin^2\frac{\omega}{2})$$

periodisation in $\tilde{U}_p$.

4. Language

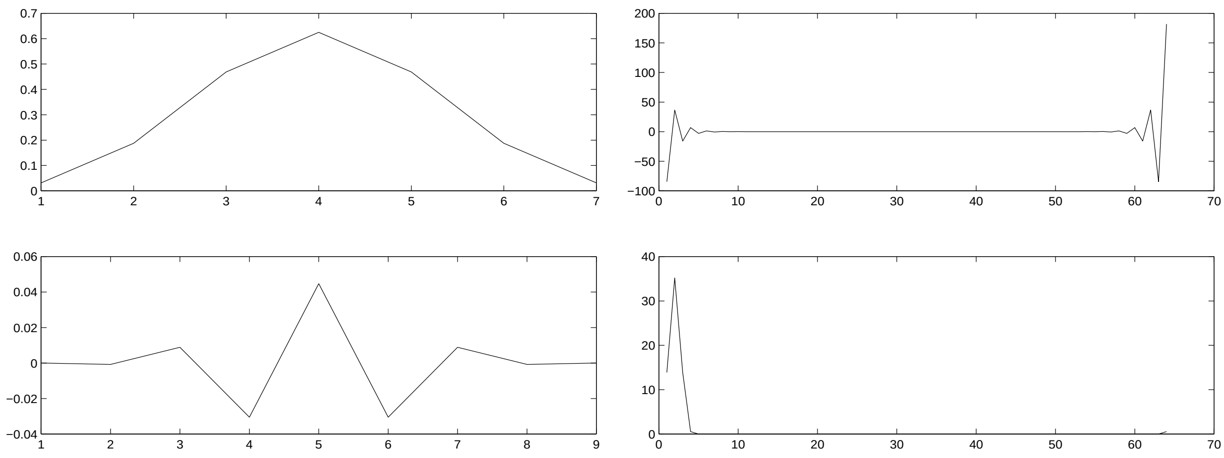
fortran77

5. Input parameters

m order of B-Splines function , p dimension of the approximation space (2^p).

6. Output parameters

$m = 4, p = 6$



Orthogonal and Biorthogonal splines filters

1. Module

filtres.f90 multiresolution analysis $(V_j, \tilde{V}_j)_j$ [?]. Filters of biorthogonal multiresolution analysis are provided by the action of operators of the form the $\frac{\partial^k}{\partial x^k}$ on the Splines orthogonal MRA.

2. Author

Guichaoua Murielle

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3. Description

filters.f90 computes the filters associated to the spline multiresolution analysis [?]. More precisely, it computes the low pass filter noted f_h, the highpass filter noted f_g and the interpolation filter noted f_i.

Moreover, filters.f90 computes filters associated to the biorthogonal multiresolution analysis obtained by application of a differential operator $\frac{\partial^k}{\partial x^k}$ on a spline wavelets base of order m [?]. These filters are noted f_{bh} for the low pass filter, f_{bg}

for the high pass filter and f_{bi} for the interpolation filter.

4. Language

   fortran 90

5. Input parameters

   Spline order (even):m
   Signal size: $N = 2^p$
   Order of the differential operator: k

6. Output parameters

7. Output parameters

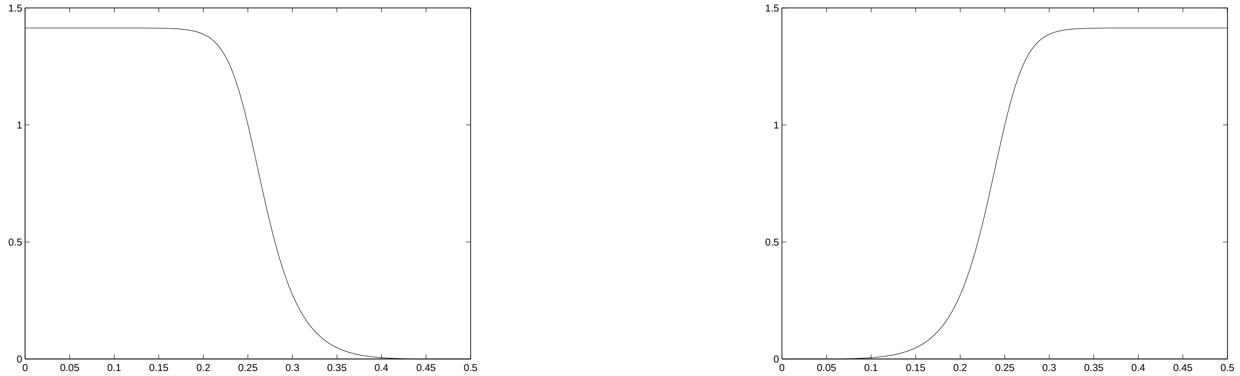


Figure 1: Filters f_h and f_g associated to the spline MRA of order $N = 4$.

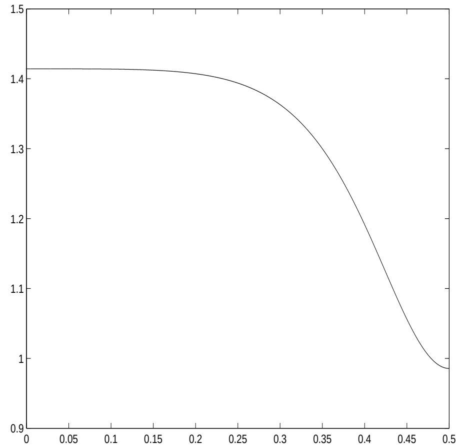
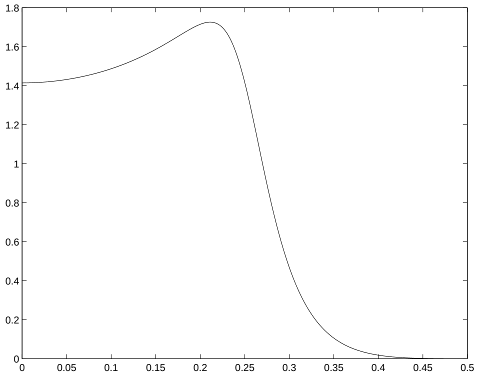


Figure 2: Interpolation filter f_i associated to the spline MRA of order $N = 4$.



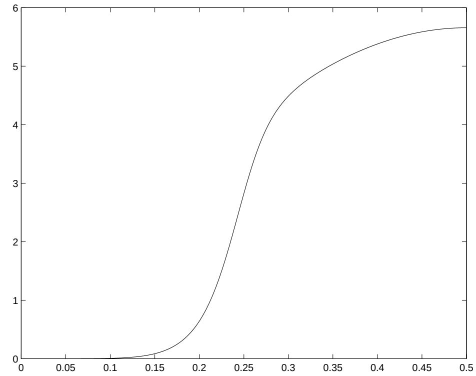


Figure 3: Filters f_{bh} and f_{bg} for spline MRA of order $N = 4$ and for the operator $\partial/\partial x$.

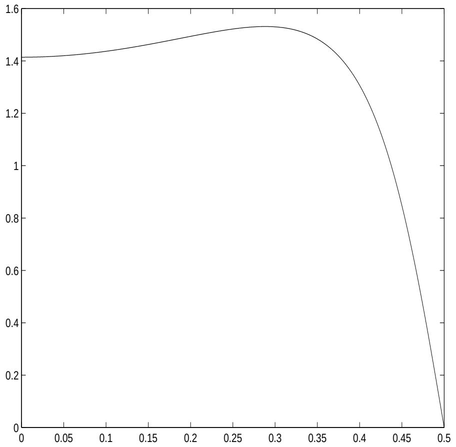


Figure 4: Filters f_{bi} for spline MRA of order $N = 4$ and for the operator $\partial/\partial x$.

MRA biorthogonal to the B-spline MRA

1. Module
   interbi.f90

2. Author
   Guichaoua Murielle
   email: guicha@marius.univ-mrs.fr
3. Description

   interbi.f90 computes scaling functions and wavelets, biorthogonal to respectively the B-spline scaling functions and the B-spline wavelets [?]. We compute these functions by using A. Harten works and interpolatory techniques [?],[?] [?]. For example, it could be Lagrange interpolations, Hermite interpolations or trigonometric interpolations. We present the case of the polynomial Lagrange interpolations of order r. In this case, this interpolation can be computed on the r-stencil defined by $\{x_{j,k-l+1}, x_{j,k-l+2}, \ldots, x_{j,k-l+r+1}\}$ whith $l = r, \ldots, 1$ For each stencil, we construct biorthogonal scaling functions noted $\tilde{\phi}_{N,r}$ and biorthogonal wavelets noted $\tilde{\psi}_{N,r}$.

4. Language
   fortran 90
5. Input parameters

B-spline order (even): m
Signal size: $N = 2^p$
Interpolation's order: r
Stencil chosen: l

6. Output parameters

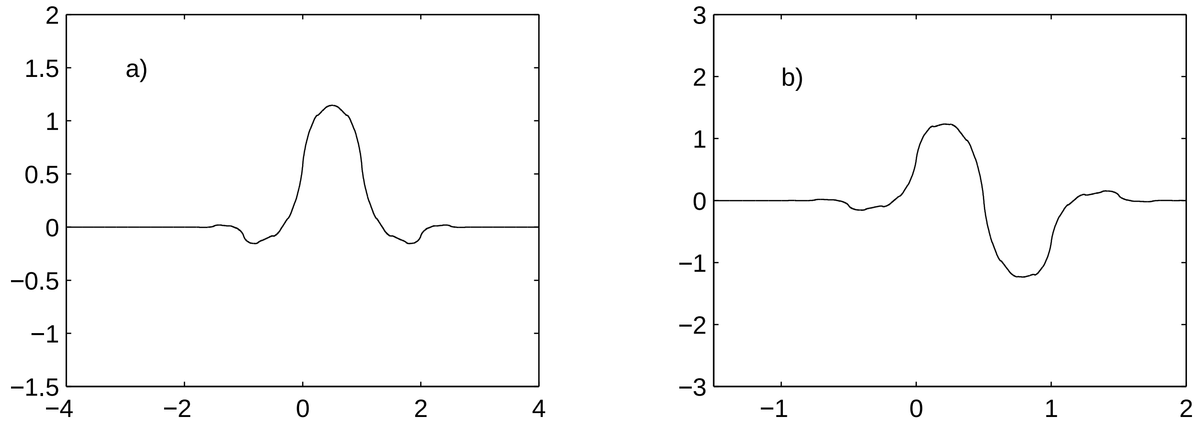


Figure 5: a) Scaling function $\tilde{\phi}_{N=1,r=3}$; b) wavelet $\tilde{\psi}_{N=1,r=3}$ for $l = 2$

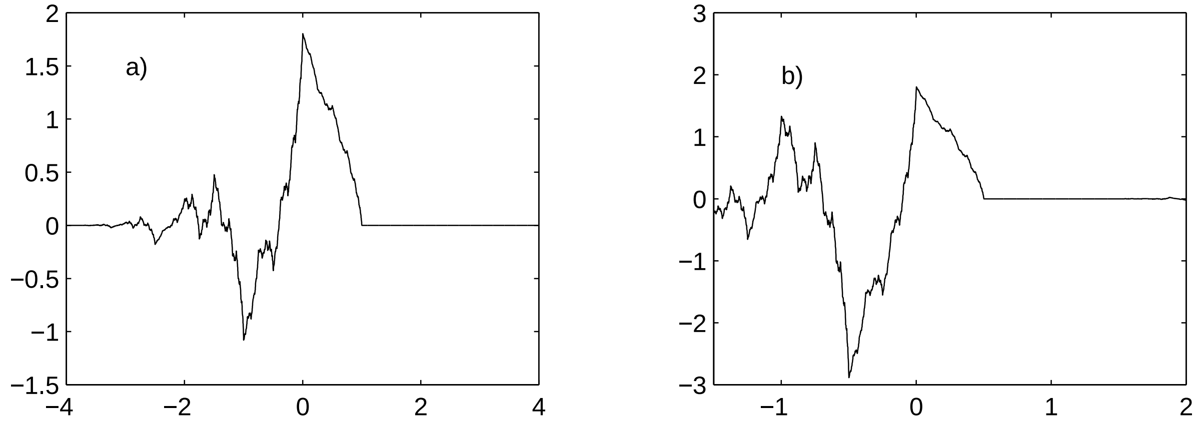


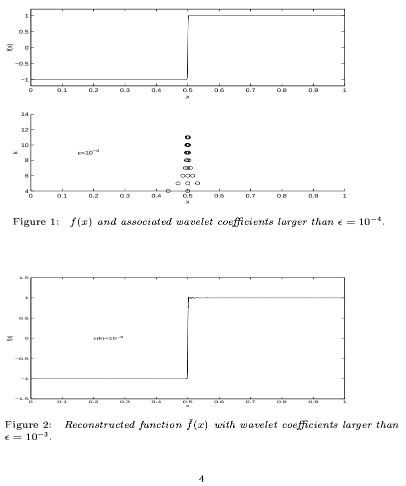
Figure 6: a) Scaling function $\tilde{\phi}_{N=1,r=3}$; b) wavelet $\tilde{\psi}_{N=1,r=3}$ for $l = 1$

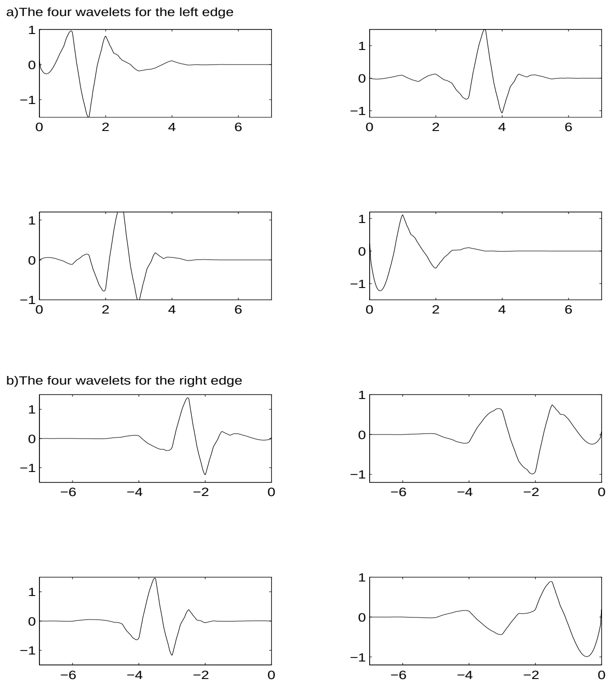
Wavelets on the Interval with Homogeneous Boundary Conditions

1. Module
   EXECUTE.c, FoncHomog3.c,....
   filters for scaling functions and wavelets on the interval with boundary conditions. Associated algorithms like tree–algorithm, thresholding algorithm, visualisation of boundary wavelets.
2. Author
   Guillaume Chiavassa
   email: guillaume.chiavassa@uv.es
3. Description
   EXECUTE.c: Main file. Call all the other functions.
   Algorithme-Arbre.c: Mallat's algorithm for wavelet on the interval.
   Init-AMR.c: Computation of mirror quadrature filters for the boundary wavelets depending of the boundary conditions.
4. Language
   C
5. Input parameters
   In the file PARAMETRE. Number of vanishing moments for the multiresolution analysis, boundary conditions on edges 0 and 1, scale for the decomposed function (V_j), and for the reconstructed (V_{j+p}). Thresholding parameter.

6. Output parameters

Values for the filters coefficients for edges 0 and 1. Values on figure





Periodic Tree Agorithms

1. Module
   trof.f
2. Author
   Jacques Liandrat
   email: `liandrat@marius.univ-mrs.fr`
3. Description
   trof.f: Performs the fast wavelet transform on periodic orthogonal wavelet bases:

   Wavelet coefficients of the interpolant $\Longleftrightarrow$ Point values

   using precalculated filters (m_0, m_1 and interpolation) computed in the routine filtre.f.
4. Language
   fortran 77
5. Input parameters

   Vector of point values or of wavelet coefficients : $\{f_i\}_{i=0,n-1}$
   Number of points: n ($n = 2^p$)
   Filter values: fh, fg, fl, 3 vectors of length $n/2+1$
   Parameter is:
   $is = 1$ Point values $\Rightarrow$ Wavelet coefficients of the interpolant
   $is = -1$ Wavelet coefficients of the interpolant $\Rightarrow$ Point values

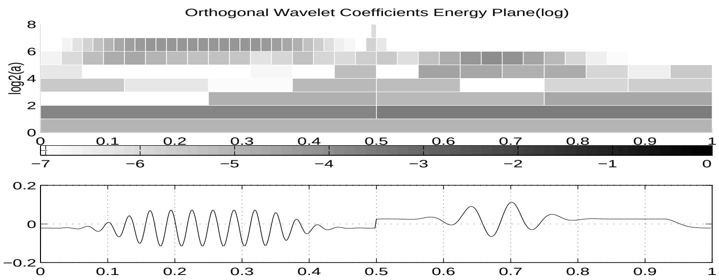
6. Output parameters

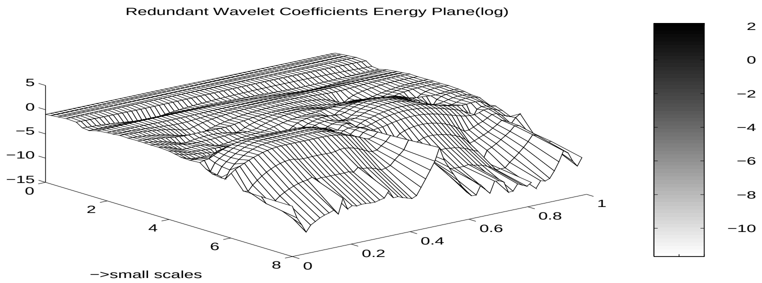
Vector of wavelet coefficients or of point values : $\{f_i\}_{i=0,n-1}$

Redundant B–spline Analysis

1. Module
   wallis.m
2. Author
   Philippe Dussouillez
   email: dussou@marius.univ-mrs.fr
3. Description
   A set of Matlab programs devoted to orthogonal and redundant periodic spline wavelet transform, computation and visualization.
   Computation of $\{T_s(\alpha, \beta) = \int s(x)\psi_\alpha(x - \beta)dx\}$.
   orthogonal case: $\alpha = 2^{-j}, j = 0, \ldots, p-1, \beta = k2^{-j}, j = 0, \ldots p - 1, 0 \leq k \leq 2^j - 1$.
   redundant case: $\alpha = \frac{2j}{2^{(p+n)}}$, $\beta = k2^{-p}$, $j, n \in \mathbb{N}$, $0 \leq k < 2^p$.
   Several graphic programs are provided.
4. Language
   Matlab
5. Input parameters

   spline order (even): m, signal: $\{s_i\}_{i=0,2^p-1}$
   scale mini: $s_{\min}$, scale maxi: $s_{\max}$
   number of intermediate scale per octave (redundant)
6. Output parameters
   wavelet matrix coefficients





7. Remarks
   A version for complex signal and wavelets is available.

Wavelet preconditioner for finite differences operator

1. Module
   precond2.f, precond2_zero.f, precond2_moy.f, precond2_moy_zero.f preconditioner for the finite differences operator L_p associated to L, $L = Id - \{\frac{\partial}{\partial x}(a(x)\frac{\partial}{\partial x})\}$. Using B-Splines BMRA

2. Author
   Anne–Sophie Piquemal
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3. Description
   precond2.f: discretisation of L on a regular grid of mesh size $h = \frac{1}{2^p}$. L_p is considered as a continuous operator on V_p.
   L_p is connected to an ill conditioned matrix M_p.
   preconditioner based on the decomposition

$$V_p = V_q \oplus (\bigoplus_{j=q}^{p-1} W_j)$$

- in V_q: inversion of $\Pi_q L_p \Pi_q^*$, where Π_q projection on V_q, to obtain A_q.

- in $\bigoplus W_j$: σ_p is the symbol of Δ_p, discretised operator corresponding to $\Delta = -\frac{\partial^2}{\partial x^2}$.

For each wavelet ψ_{jk}, $j \geq q$:

– From localisation properties of wavelets, approximation of $L_p^{-1}\psi_{jk}$ by $\frac{1}{a(x_{jk})}\Delta_p^{-1}\psi_{jk}$ where $a(x_{jk})$ value of the coefficient at the middle of the wavelet support.

– D_1:$\Delta_p^{-1}\psi_{jk} \rightarrow \frac{1}{\sigma_p(2^j)}\tilde{\theta}_{jk}$ is bounded, where $\tilde{\theta} = \Delta^{-1}\psi$ corresponds to MRA $(\tilde{U}_j)_j$ described previously.

– D_q: $L_p^{-1}\psi_{jk} \rightarrow \frac{1}{a(x_{jk})}\frac{1}{\sigma_p(2^j)}\tilde{\theta}_{jk}$ is bounded.

$A_q + D_q$ preconditioner for L_p, corresponds to a matrix C_p.

precond2_zero.f: idem with $q = 0$
precond2_moy.f, precond2_moy_zero.f: idem with $a(x_{jk})$ defined as the average of the coefficient on the wavelet support, and $q = 0$.

4. Language
   fortran77

5. Input parameters
   order of the B-Splines m, p dimension of the approximation space (2^p), and order of the decomposition q.
   choice of the variable coefficient $a(x)$.

6. Output parameters
   $q = 0$, $a(x_{jk})$ value at the middle of the wavelet support

p	$x(x-1)+1$	$2x(1-x)^2+\frac{1}{2}$	$2+cos^2(2\pi x)$
3	12.149	66.689	3.325
4	11.713	74.521	3.400
5	11.668	82.941	3.447
6	12.020	85.720	3.460
7	12.118	86.508	3.463
8	12.144	86.722	3.464
9	12.151	86.778	3.464

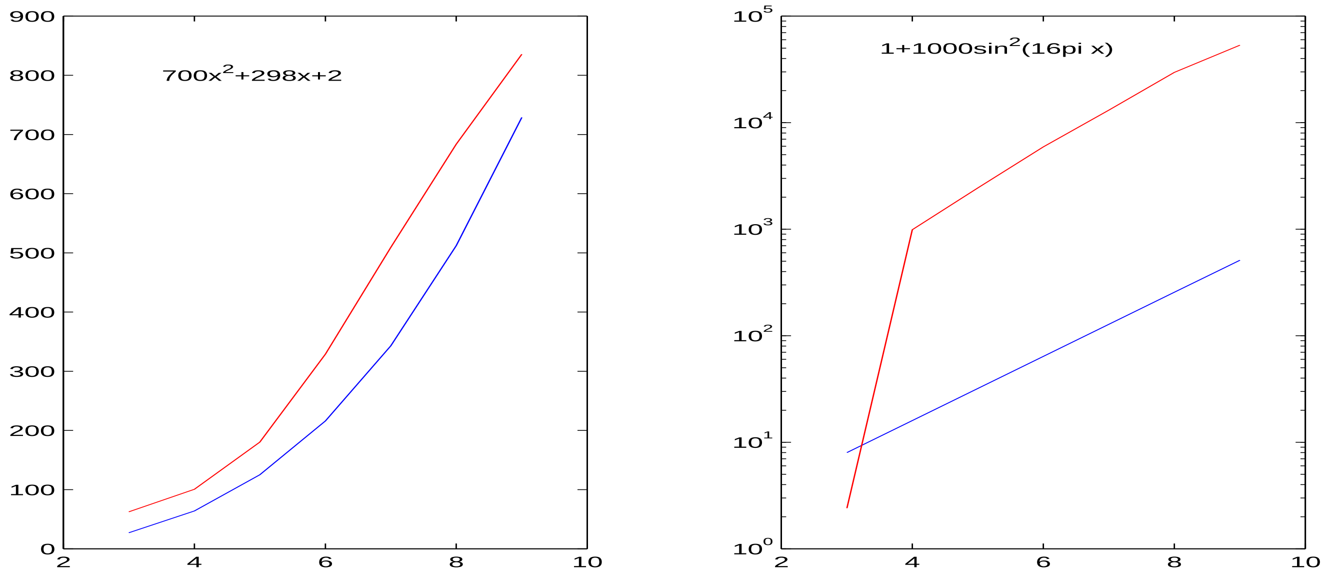
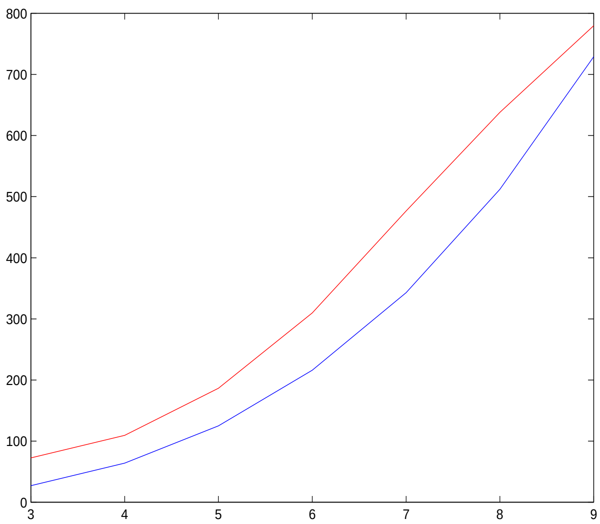


Figure 7: $K(C_pM_p)$ versus p

for the "smooth" coefficients $K(C_pM_p)$ is in $O(1)$.
for $a(x) = 700x^2 + 298x + 2$, $K(C_pM_p)$ is in $O(p^3)$
for $a(x) = 1 + 1000sin^2(16\pi x)$, $K(C_pM_p)$ is in $O(2^p)$.

$q = 0$, $a(x_{jk})$ average on the wavelet support

p	$x(x-1)+1$	$2x(1-x)^2+\frac{1}{2}$	$2+cos^2(2\pi x)$
3	17.234	171.316	2.688
4	16.616	165.077	2.995
5	16.465	165.020	3.156
6	16.427	180.299	3.026
7	16.418	190.361	2.9671
8	16.416	196.186	2.950
9	16.550	199.063	2.9466



p	$1+1000sin^2(16\pi x)$
3	1205.95
4	2.412
5	2.446
6	9.941
7	43.533
8	146.311
9	250.480

Table 1: $K(C_pM_p)$ versus p

for the "smooth" coefficients $K(C_pM_p)$ is in $O(1)$.
for $a(x) = 700x^2 + 298x + 2$, $K(C_pM_p)$ is in $O(p^3)$
for $a(x) = 1 + 1000sin^2(16\pi x)$, large improvement of the result

Comparison with another preconditioners

1. Module
   bpx1.f, bpx1_moy.f, yser.f
   preconditioners for finite differences operator L_p associated to L, $L = Id - \{\frac{\partial}{\partial x}(a(x)\frac{\partial}{\partial x})\}$. Using B-Splines BMRA

2. Author
   Anne–Sophie Piquemal
   email: piquemal@marius.univ-mrs.fr

3. Description
   bpx1.f:([1]) preconditioner constructed by J.H Bramble, J.E Pasciak and J. Xu.
   Based on the definition of Π_q projection on V_q, BPX preconditioner is given by: $B_p = \sum_{k=0}^{p} \frac{1}{\lambda_k}\Pi_k$, where $\lambda_k = \rho(\Pi_k M_p \Pi_k^*)$, spectral radius.
   bpx1_moy.f: idem but we approximate λ_k by 4^k.

   yser.f:([2]) preconditioner constructed by H. Yserentant.
   Diagonal preconditioner in the hierarchical basis.

4. Language
   fortran77

5. Input parameters
   order of B-Splines function m, p dimension of the approximation space (2^p), variable coefficient $a(x)$.

6. Output parameters
   Comparison in terms of preconditioning, CPU times (differences between BPX with the exact spectral radius and BPX with the approximation)

7. References

[1] J.H Bramble, J.E Pasciak and J. Xu.*Parallel Multilevel Preconditioners.* Math. Comp. Nr. 55, pp. 1-22, 1990

[2] H. Yserentant. *On the multilevel splitting of finite element spaces* Num. Mathematik. Nr. 58, pp. 163-184, 1990

Explicit wavelet scheme for evolution equations

1. Module
   dtcons.f90

2. Author
   Guichaoua Murielle
   email: `guicha@marius.univ-mrs.fr`

3. Description

   dtcons.f90 computes the solution of equation written under the form $\frac{\partial u}{\partial t} = F(u)$ where F is a non linear operator $F(u) = -D(f(u)) + L(u)$ with L and D, two differential operators and f, a non linear fonction [?] [?].
   Using Taylor scheme, the scheme is written:

$$\frac{u^{n+1} - u^n}{\Delta t} = F(u^n) + \frac{\Delta t}{2} F^{[1]}(u^n) + \ldots + \frac{\Delta t^{r-1}}{r!} \quad F^{[r-1]}(u^n) \tag{1}$$

   où $F^{[0]} = F$ et $F^{[l+1]} = D_t F^{[l]} + D_y F^{[l]}.F$.

   If $u^n \in V_p$ where $(V_j)_j$ is a spline MRA, $u^n(x) = \sum_{j \le p-1,k} d^n_{j,k} \psi_{j,k}(x)$.

   Then the scheme can be written:

$$\sum_{j,k} \frac{d_{j,k}^{n+1} - d_{j,k}^{n}}{\Delta t} \psi_{j,k} = F(\sum_{j,k} d_{j,k}^{n} \psi_{j,k}) + \frac{\Delta t}{2} F^{[1]}(\sum_{j,k} d_{j,k}^{n} \psi_{j,k}) + \ldots + \frac{\Delta t^{r-1}}{(r)!} F^{[r-1]}(\sum_{j,k} d_{j,k}^{n} \psi_{j,k}). \quad (2)$$

In the case where $F(u) = -\frac{\partial u}{\partial x}$:

$$u^{n+1}(x) = \sum_{l=0}^{r} \frac{(-\Delta t)^l}{l!} \sum_{j \leq p-1,k} < u^n, \psi_{j,k} > L^l \psi_{j,k}. \quad (3)$$

For each l we had to compute the filters f_h, f_g, f_l, f_{bh}, f_{bg} and f_{bi}.

Numerical analysis of the scheme

We show that the scheme (3) is stable under the condition:

$$\exists\ C, 0 < C < +\infty, \quad \text{such that} \quad \Delta t 2^{2p} \leq C. \quad (4)$$

Under the stability condition (4) and assuming that p is the smallest integer such that $V \subset V_p$, the following convergence estimate holds: $\exists\ M, 0 < M < +\infty,$ such that

$$\| E_V^n \| = \| u^n - \Pi_V u(x, t_n) \|_2 \leq M(2^{-pm} + \Delta t^r) \quad (5)$$

4. Language
   fortran 90
5. Input parameters

Spline order (even): m
Signal size: $N = 2^p$
Time's order scheme: k
Time step: Δt
Maximal time: tmax

6. Output parameters

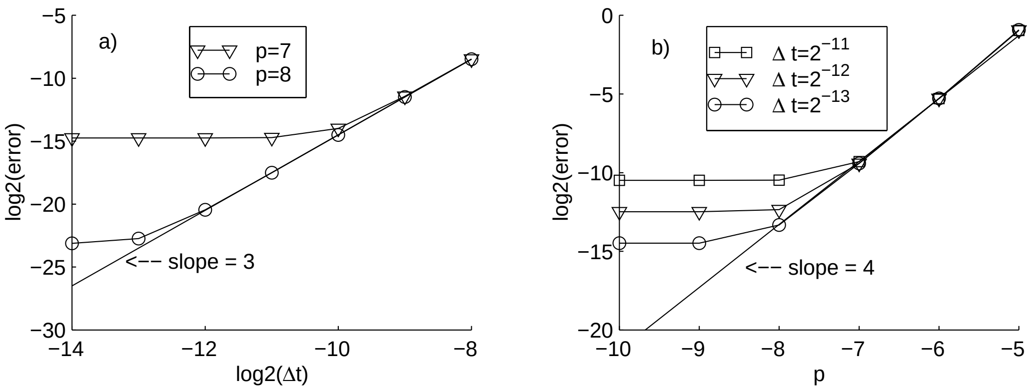


Figure 8: Error E_V^n at $tmax = 0.1$ for a) a time scheme of order $r = 3$, p is fixed at $p = 7$ and $p = 8$, $m = 8$, b) a time scheme of order $r = 2$ and $m = 4$, Δt is fixed at 2^{-11}, 2^{-12} and 2^{-13}.

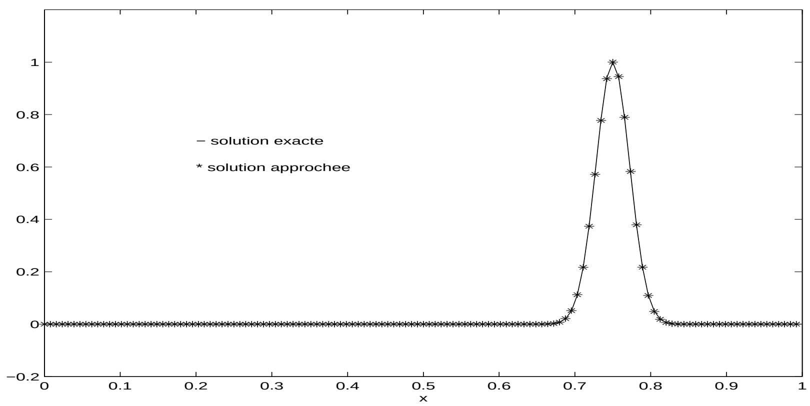


Figure 9: Wavelet scheme for the resolution of $\frac{\partial u}{\partial t} + \frac{\partial u}{\partial x} = 0$ with $u(t, 0) = exp(-(x - 0.25)^2 * 1000)$ at time $t = 0.1$ with $N = 2^7$, $\Delta t = 0.1\Delta x$ and $m = 4$.

Explicit wavelet scheme for evolution equations with variable time step

1. Module
   dtvar.f90

2. Author Guichaoua Murielle
   email: guicha@marius.univ-mrs.fr

3. Description

   dtvar.f90: The stability condition $\Delta t 2^{2p} \leq C$ leads to introduce a multiscale algorithm for the time discretization. It is, shortly speaking, constructed by applying the above described scheme to every scale of the approximation with a scale dependant time step [?].

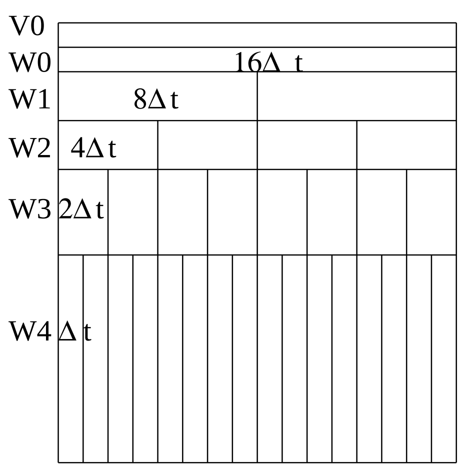


Figure 10: Adaptive grid for $p = 5$.

4. Language
   fortran 90

5. Input parameters

   Spline order (even): m
   Signal size: $N = 2^p$
   Time's order scheme: k
   Time step: Δt
   Maximal time: tmax

6. Output parameters

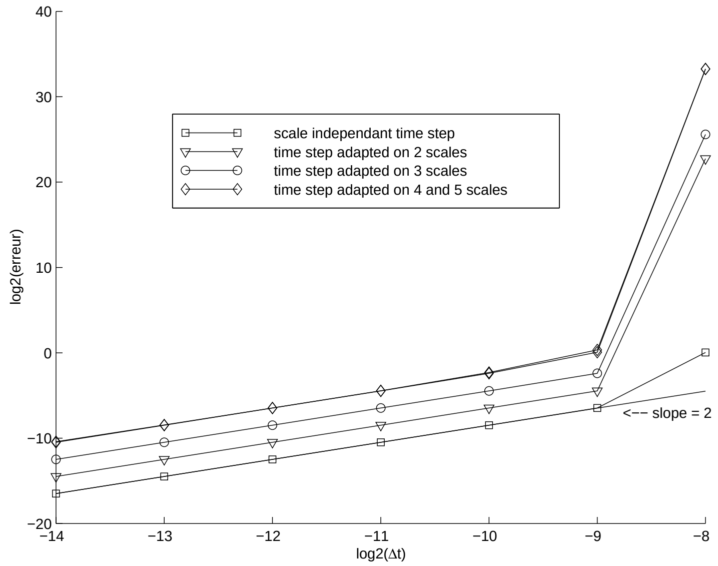


Figure 11: Error E_V^n for a time scheme of order $r = 2$ at $tmax = 0.1$. The resolution scale is V_8, the time step Δt is taken from 2^{-8} to 2^{-14} and spline order is fixed at $m = 8$.

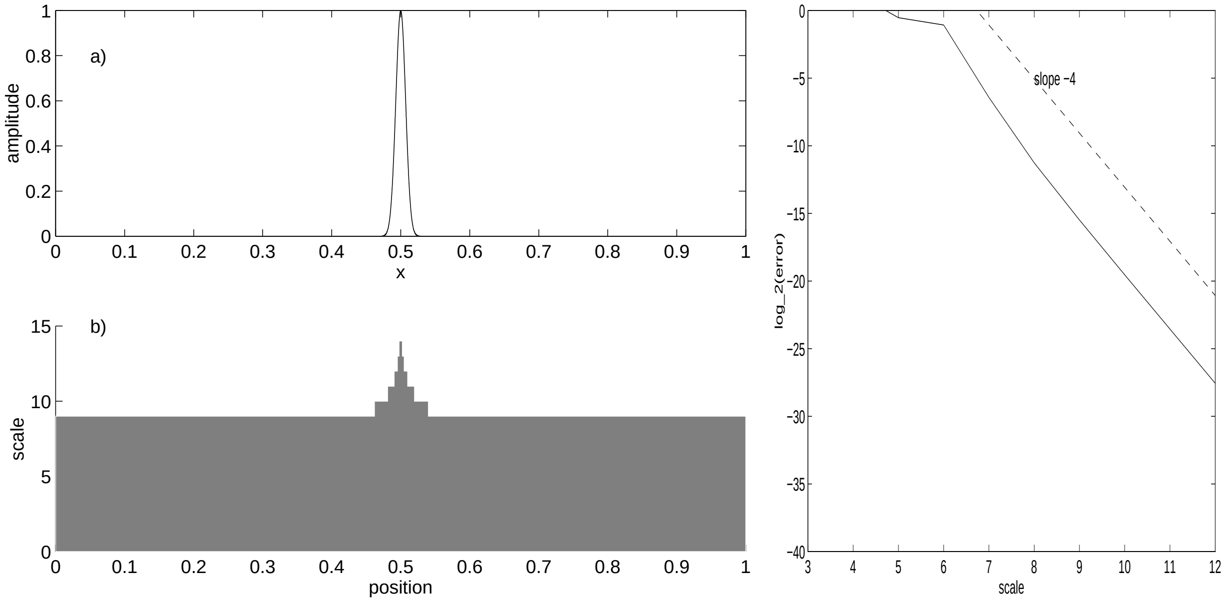
Evaluation of nonlinear functionals in wavelet basis

1. Module
   nonlin.f
2. Author
   Jacques Liandrat
   email: liandrat@marius.univ-mrs.fr
3. Description
   nonlin.f: Evaluation of nonlinear functional (Ex: $f(u) = u^\alpha$) on a "non regular c–structured " wavelet space. Numerical estimate of the errors for simple cases.
4. Language
   fortran77
5. Input parameters

   Index of the smallest scale of the space of approximation: p
   Index of the smallest "full" scale: q
   u, f: defined analytically
   Wavelet analysis filters: defined in filtre.f

6. Output parameters

   Wavelet coefficients of the approximation of $f(u)$
   Error estimates



7. References

[1] J. Liandrat, Ph. Tchamitchian. *On the fast approximation of some non linear operators in non regular wavelet space.* Advances in computational Mathematics Nr. 8, pp. 179–192 , 1998

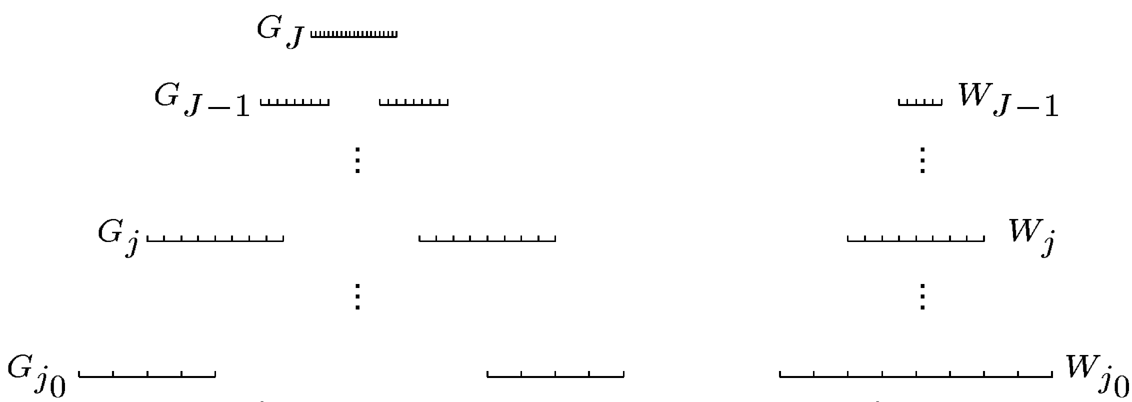
Adaptive Wavelet Spaces on $[0,1]^2$

1. Module
   Adaptive 2D
   adaptive multiscale transformation on $[0,1]^2$

2. Author
   Jürgen Vorloeper
   e–mail: jvor@irphe.univ-mrs.fr

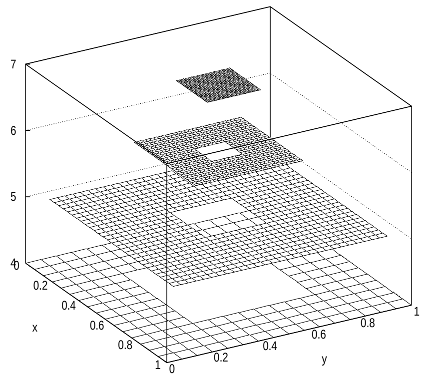
3. Description
   Representation of a function f in wavelet basis, orthogonal spline wavelets, periodic b.c. Basdevant/Perrier 89

   - adaptive spaces in 1D:

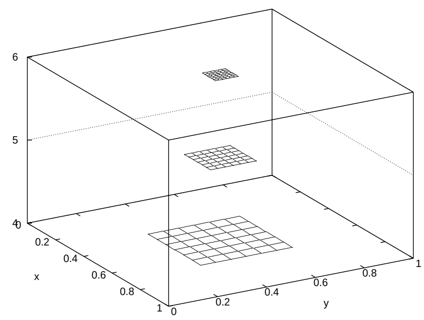


a) generators b) wavelets

   - 2D: tensor products



a) generators



b) wavelets

- general situation: set of cones, stretched in x or y direction, distinct number of levels

Multiscale Transformation on a given grid, filter $\mathbf{h}^{(j)}$, $\mathbf{g}^{(j)}$ for generators/wavelets

$$f = \sum_{k \in \Delta_J} c_{j,k}\, \phi_{j,k} = \sum_{j=j_0}^{J-1} \sum_{k \in \nabla_j} d_{j,k}\, \psi_{j,k}$$

- point evaluation

$$c_{j,k} \doteq 2^{-j/2} f(2^{-j} k) \qquad \forall k \in G_j, j = j_0, \ldots, J$$

- decomposition

$$c_{j-1,k} = \sum_{r=-N}^{N} c_{j,n+2r}\; h_r, \qquad d_{j-1,k} = \sum_{r=-N}^{N} c_{j,n+2r}\; g_r \tag{6}$$

- reconstruction

$$c_{j,k} \doteq \sum_{r=-N}^{N} c_{j-1,\frac{k-r}{2}} \, h_r + \sum_{r=-N}^{N} d_{j-1,\frac{k-r}{2}} \, g_r \qquad (7)$$

$\rightsquigarrow$ direct application of (1), (2)
$\rightsquigarrow$ FFT $\longrightarrow$ multiplication $\longrightarrow$ inv_FFT

4. Language
   C++

5. Input Parameters
   function f, position/shape of cones, level j, J, filter $\mathbf{h}^{(j)}$, $\mathbf{g}^{(j)}$ of length $\leq N$

6. Output Parameters
   access to scaling/wavelet coefficients, point values

7. Remarks

   - some features are not yet implemented
   - routine for 2D–FFT from "Numerical Recipes in C"
   - adaptive spaces for 1D, applications, programs in C Chiavassa 97

8. References

G. Chiavassa and J. Liandrat. Fully adaptive wavelet algorithm for parabolic partial differential equa

One dimensional adaptive wavelet code for the resolution of semi–linear parabolic equations

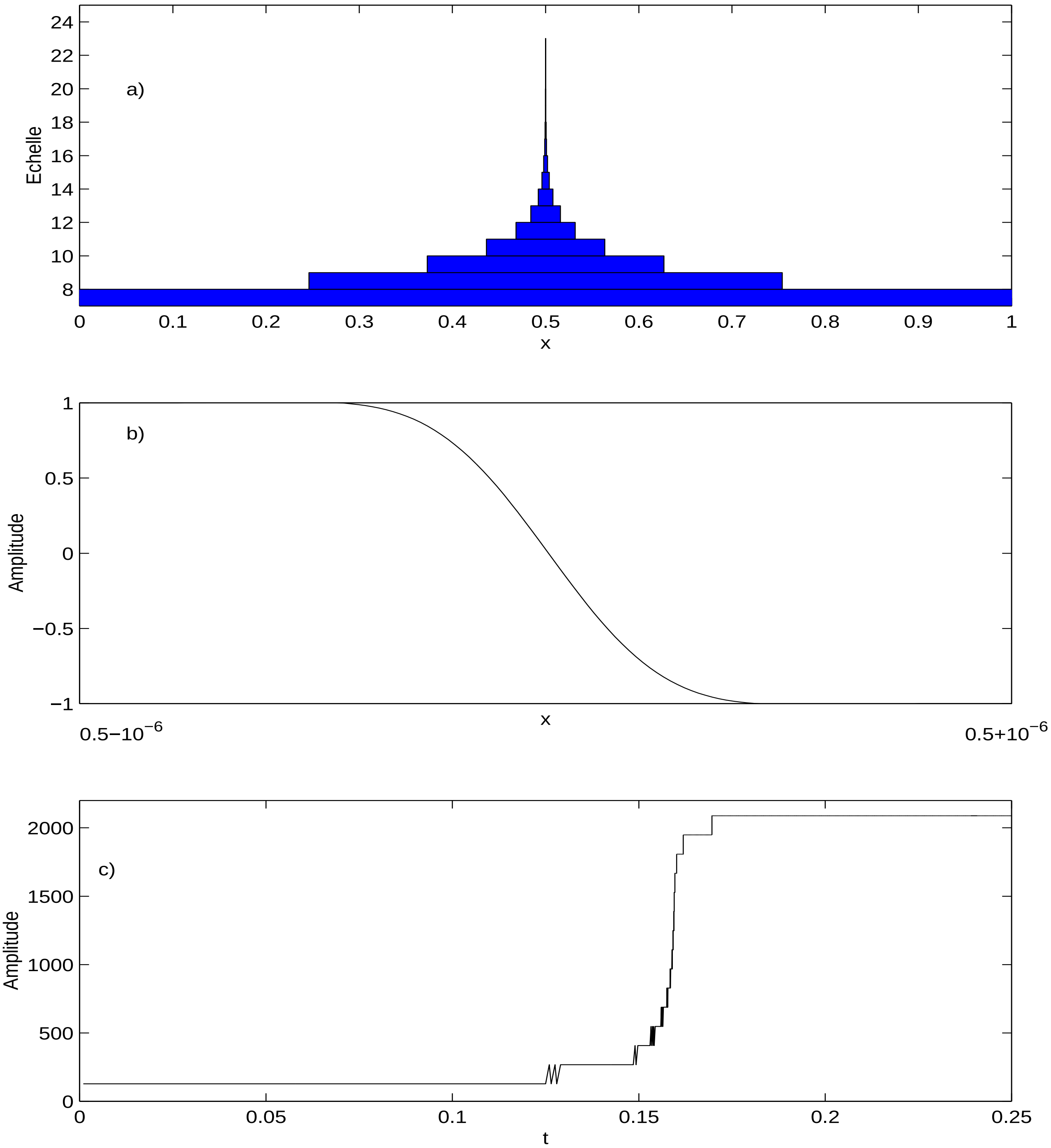
1. Module
   Adaptatif,Burgers,Flamme,Chaleur,NLchaleur,Adaptime
   Directories corresponding to the different applications of the wavelet adaptive code
2. Author
   Chiavassa Guillaume
   email: guillaume.chiavassa@uv.es
3. Description
   Adaptatif: files corresponding to adapted tree-algorithm, computation of non-linear terms on a non-regular grid, evaluation of filters corresponding to the homogeneous operator (heat operator for ex.), application of this operator to function defined on an adapted wavelet space.
   Burgers: applications of the programs defined in Adaptatif to the 1D Burgers equation.
   Flamme: applications of the programs defined in Adaptatif to the 1D reaction-diffusion equation governing the evolution of a planar flamme.
   Chaleur: applications of the programs defined in Adaptatif to the 1D Heat equation.
   NLchaleur: applications of the programs defined in Adaptatif to the 1D non-linear heat equation.

Adaptime: applications of the programs defined in Adaptatif to the 1D Burgers equation with adaptation of the time step.

4. Language
   C

5. Input parameters
   Depends of the considered test case. General parameters are lower level of the approximation space (V_{j_0}), thresholding parameters, computational time,...

6. Output parameters
   Values of the computed solution, representation of the adapted space of approximation,...


a)
Echelle
24
22
20
18
16
14
12
10
8
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
x
b)
Amplitude
1
0.5
0
−0.5
−1
x
0.5−10^−6
0.5+10^−6
c)
Amplitude
2000
1500
1000
500
0
0
0.05
0.1
0.15
0.2
0.25
t