A lagrangian discontinuous Galerkin method for unstructured 2-dimensional meshes

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The numerical metho is based on the moments of the conservation equations with or without sources, for an unstructured triangular mesh.

It is supposed that the physical quantities are represented by polynomial functions of order n. They are piecewise continuous and their moments are obtained by using Bernstein polynomials of order n, in each cell.

One consider as well the moments of the Dirac lines on the cell contours.

The cell nodes are defined by their lagrangian coordinates which are time independent.

This define a hierarchy of numerical schemes where each element corresponds to an order of precision in space.

It is easy to use schemes of various precision corresponding to differents parts of the mesh.

The numerical results to be presented were obtained with a scheme of 3^{rd} in space and the time integration was performed with a Runge-Kutta method of second order.

Numerical stability is guaranted by two diffusion processes. The first is based on a criteria of positivity due to the moments. This diffusion is sufficient to assure stability in absence of strong shocks.

In the case of strong shocks a second process is added which limits the spatial derivatives.

The requirement of positivity of the Jacobian (of the transformation between the eulerian and the lagrangian mesh) might require a reconstruction of the mesh. This is achieved by introducing particles which are given a weight corresponding to the physical quantity as well as a "volume" weight.

The mesh reconstruction is done using the moments of the weights assigned to the the particles. The method is conservative and little dissipative.

The lagrangian character of the scheme permits the treatment of multifluid problems without modifications if initialy each cell contains only one single fluid. However, it is possible to extend the method as to trat several fluids per cell.

Compairing the results obtained for hydrodynamic problems such as strong shocks, isentropic compressions and laser-plasma interaction, with know analytical solutions confirms the precision of the method.

The shock geometry and free surfaces are correct in un-structured meshes and numer. The method exhibits only weak numerical oscillations.

The extension to 3D poses no particular theoretical problem.