

# Renormalized Mesh-free Scheme: Convergence for Scalar Conservation Laws and Applications

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We present an analytical and numerical study of the weak renormalized mesh-free numerical scheme applied to scalar conservation laws. Renormalization is a tool introduced in order to eliminate the Smoothed Particle Hydrodynamics (SPH) lack of consistency of first order derivatives. The weak renormalized scheme, is derived from the general conservation laws weak formulation and turns out to be conservative. This scheme being unstable, we introduce a numerical viscosity in analogy with the Finite Volume method, and we establish the convergence of the scheme discretized in time. We will show that the techniques developed in the case of SPH convergence proof can not be applied to the renormalized scheme because of the new form of the derivative approximation. Several applications of the renormalized mesh-free scheme will be presented, especially in the case of multimaterial systems including damage models.

**Key Words :** mesh-free methods, SPH, renormalization, weak scheme, pseudo-viscosity, Kruskov entropy method, convergence, damage models.