Towards GrAALE: error estimation and grid adaptation

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A true physics based predictive capability for complex systems requires to handle multiple length and time scales. A key element is the ability to resolve different scales locally using adaptive grid spacing. Adaptivity can be achieved by moving mesh methods or by adaptive mesh refinement. Either way guidance must be available on where more grid points and more computational effort has to be focused. It follows that an accurate determination of the local accuracy of the solution to a given problem is required.

The ultimate goal of the present effort is to bring the issue of practical determination of the local accuracy to a new level of sophistication. Traditionally, the issue of a posteriori error determination has been attacked in one of two ways. First, the highly theoretical approach of estimating the error based on very complex mathematical arguments. Secondly, the empirical and heuristic use of indicators that are generally problem dependent and based on the physical intuition of the user. My approach is to combine the mathematically soundness of the more theoretical approach with the practical effectiveness of the heuristic approach.

This effort has led to the development of a new grid adaptation strategy based on an *operator recovery error origin detector*. This approach has been inspired by the abundant literature on a posteriori error estimation in the area of finite elements and particularly of moving finite element. The approach is to use interpolation recovery of the discretized operators typical of the problem at hand and of the solution functions upon which the operators act. Based on the interpolation recovery, the error in the solution can be obtained comparing the exact operators with the recovery of the discretized operators. Grid adaptation can be based on the operator recovery error origin detector using the variational approach.

The operator recovery grid adaptation has been tested in many applications ranging from elliptic problems typical of plasma physics and computational electrodynamics, such as the Debye and Poisson equations, to hyperbolic problems in gas dynamics, neutron transport and plasma physics.

In summary, I will present a new approach for grid adaptation based on the operator recovery error origin detector, showing its superiority in a number of applications. The primary advantages are:

1. Accuracy much superior to heuristic error indicators.
2. The new approach is automatic, not requiring any user intervention or any previous knowledge of the physics of the problem at hand.
3. Reliability; the new approach has been tested in a large variety of problems, never failing: in all cases considered it was always superior to approaches based on other indicators currently widely used.
4. Simplicity; the new approach is simple, the operator recovery approach can be easily devised for any given problem and can be implemented straightforwardly in any existing code. I have so far applied it to Lagrangian, ALE and Eulerian codes using structured and unstructured meshes, including the AMR code CLAWPACK.