A Two Fluid Model for Two-Phase Flows with Free Interface

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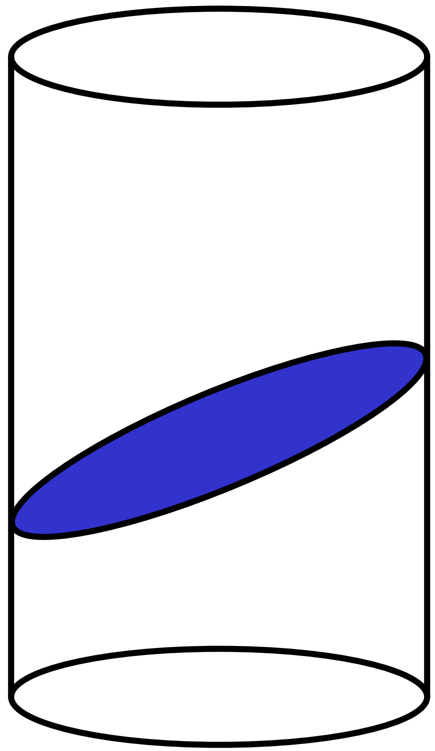
Context and Applications

Fluids behaviour in tanks :

- Sloshing effects due to transverse accelerations
- Surface tension effects due to possible gravity reduction and interface curvature
- Thermal effects due to external heat fluxes



Main Features of our approach

- Eulerian method on a fixed grid, with no interface reconstruction or interface tracking, existence of an artificial mixture zone
  → need of a mixture closure law leading to a well-posed problem (hyperbolic system, thermodynamic consistency)
  → need of a low diffusive numerical scheme

- able to deal with a wide range of applications relative to fluid behaviour in a launcher tank (including coupling between thermal and hydrodynamic effects)
  → compressible two fluid model, eventually with “pseudo” sound speeds, to overcome low Mach number difficulties

Modeling Hypothesis

- Gas of density ρ_g , liquid of density ρ_ℓ, supposed to be compressible and inviscid

- no thermal effects : linearized EOS

$$p_g(\rho_g) = p_0 + c_g^2(\rho_g - \rho_{g0}) \ , \ p_\ell(\rho_\ell) = p_0 + c_\ell^2(\rho_\ell - \rho_{\ell 0})$$

- the two fluid "fictitiously" coexist everywhere, presence characterised by volume fraction α_g , α_ℓ : $\alpha_g + \alpha_\ell = 1$

- only one velocity field

The Equilibrium Model

$$(E)\begin{cases}\dfrac{\partial\tilde{\rho}_g}{\partial t}+\dfrac{\partial}{\partial x}\tilde{\rho}_g u+\dfrac{\partial}{\partial y}\tilde{\rho}_g v=0\\[2mm]\dfrac{\partial\tilde{\rho}_\ell}{\partial t}+\dfrac{\partial}{\partial x}\tilde{\rho}_\ell u+\dfrac{\partial}{\partial y}\tilde{\rho}_\ell v=0\\[2mm]\dfrac{\partial}{\partial t}\rho u+\dfrac{\partial}{\partial x}\left(\rho u^2+P\right)+\dfrac{\partial}{\partial y}\rho uv=\mathbf{F}_x\\[2mm]\dfrac{\partial}{\partial t}\rho v+\dfrac{\partial}{\partial x}\rho uv+\dfrac{\partial}{\partial y}\left(\rho v^2+P\right)=\mathbf{F}_y\end{cases}$$

with: $\tilde{\rho}_g=\alpha\rho_g$, $\tilde{\rho}_\ell=(1-\alpha)\rho_\ell$, $\rho=\tilde{\rho}_g+\tilde{\rho}_\ell$

The Equilibrium Model : closure laws
mixture pressure

<u>Mixture pressure P :</u>

$$P(\tilde{\rho}_g, \tilde{\rho}_\ell) = \alpha^* \, p_g\left(\frac{\tilde{\rho}_g}{\alpha^*}\right) + (1-\alpha^*)\, p_\ell\left(\frac{\tilde{\rho}_\ell}{1-\alpha^*}\right)$$

where :

$$\alpha^* = \alpha^*(\tilde{\rho}_g, \tilde{\rho}_\ell)$$

is the equilibrium gas volume fraction insuring :

$$p_g\left(\frac{\tilde{\rho}_g}{\alpha^*}\right) = p_\ell\left(\frac{\tilde{\rho}_\ell}{1-\alpha^*}\right)$$

The Equilibrium Model : closure laws
source terms

Source terms **F** :

- acceleration in the local reference frame
  (gravity + transverse acceleration)

- surface tension, CSF way : $\mathbf{F}_C = \sigma\kappa\nabla\alpha$
  (Brackbill *et al*, JCP 1992)

σ the surface tension coefficient
κ the interface curvature with $\kappa\nabla\alpha = -\nabla\cdot\left(\frac{\nabla\alpha}{\|\nabla\alpha\|}\right)\nabla\alpha$

The Equilibrium Model : closure laws

"conservative" capillary force

joint work with D. Jamet, CEA

The capillary force : $$\mathbf{F}_C = -\sigma\nabla \cdot \left(\frac{\nabla\alpha}{\|\nabla\alpha\|} \right) \nabla\alpha$$

also reads : $$\mathbf{F}_C = -\sigma\nabla \cdot \left(\frac{\nabla\alpha}{\|\nabla\alpha\|} \otimes \nabla\alpha \right) + \sigma\nabla\left(\|\nabla\alpha\|\right)$$

→ yields a completely conservative formulation of momentum equation

→ numerically, $\nabla\alpha$ needs only to be computed at the faces center

The Equilibrium Model :
mathematical properties (without surface tension)

- The mixture pressure law leads to an hyperbolic system with mixture sound speed c_* such that :

$$\rho c_*^2 = \tilde{\rho}_g c_g^2 \left(1 + \frac{K}{\alpha^*}\right) + \tilde{\rho}_\ell c_\ell^2 \left(1 - \frac{K}{1-\alpha^*}\right) \quad \text{with} \quad K = -\left(\tilde{\rho}_g \frac{\partial \alpha^*}{\partial \tilde{\rho}_g} + \tilde{\rho}_\ell \frac{\partial \alpha^*}{\partial \tilde{\rho}_\ell}\right)$$

- Existence of a Lax entropy $\mathcal{F}^*(\tilde{\rho}_g, \tilde{\rho}_\ell, \rho u) = \mathcal{F}(\alpha^*, \tilde{\rho}_g, \tilde{\rho}_\ell, \rho u)$ with :

$$\mathcal{F}(\alpha, \tilde{\rho}_g, \tilde{\rho}_\ell, \rho u) = \underbrace{\tilde{\rho}_g \int \frac{p_g}{\rho_g^2} d\rho_g + \tilde{\rho}_\ell \int \frac{p_\ell}{\rho_\ell^2} d\rho_\ell}_{\text{free energy}} + \underbrace{\frac{1}{2} \rho u^2}_{\text{kinetic energy}}$$

- Difficulties with the explicit solution of the Riemann problem due to the complexity of the mixture pressure law

$\rightarrow$ Relaxation model (in the spirit of Coquel & Perthame, SIAM 1998)

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The Relaxation Model

$$(R_\varepsilon)\begin{cases} \dfrac{\partial \alpha}{\partial t} + u\dfrac{\partial \alpha}{\partial x} + v\dfrac{\partial \alpha}{\partial y} = \dfrac{p_g - p_\ell}{\varepsilon} \quad , \ (0 < \varepsilon << 1) \\ \dfrac{\partial w}{\partial t} + \dfrac{\partial \mathrm{F}}{\partial x} + \dfrac{\partial \mathrm{G}}{\partial y} = \mathrm{S} \end{cases}$$

with $P(\alpha, \tilde{\rho}_g, \tilde{\rho}_\ell) = \alpha\, p_g\left(\dfrac{\tilde{\rho}_g}{\alpha}\right) + (1-\alpha)\, p_\ell\left(\dfrac{\tilde{\rho}_\ell}{1-\alpha}\right)$

similar models in :

- Saurel & Abgrall (JCP 1999), Saurel & Lemetayer (JFM 2001)
- Dellacherie & Rency (preprint 2001)

Remark : the relaxation source term is linked to $\mathcal{F}$ by :

$$\frac{p_g - p_\ell}{\varepsilon} = -\frac{1}{\varepsilon}\frac{\partial \mathcal{F}}{\partial \alpha} = -\frac{1}{2\varepsilon}\left(\frac{\partial \mathcal{F}}{\partial \alpha_g} - \frac{\partial \mathcal{F}}{\partial \alpha_\ell}\right)$$

thermodynamic potential

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The Relaxation Model :
mathematical properties (without surface tension)

- Also hyperbolic with mixture sound speed c such that :

$$\rho c^2 = \tilde{\rho}_g c_g^2 + \tilde{\rho}_\ell c_\ell^2$$

and verifying the sub-characteristic condition :

$$c_*^2 \leq c^2$$

→ good mathematical frame for relaxation
a Chapman-Enskog like expansion of the relaxation
model can formally be derived (as in Coquel & Perthame, SIAM 1998)

- F is an entropy, compatible with the relaxation source term :
the relaxation source term contributes to the decrease in entropy

Numerical Method

- Finite Volume method :
  - RK2 in time (2nd order)
  - Godunov-**MUSCL** in space (2nd order)

- Each RK2 stage divided in two parts
  - hyperbolic step (**exact Riemann solver**)
  - pressure relaxation step

Numerical Method : hyperbolic step

Given α^n and w^n, we solve :

$$\begin{cases} \dfrac{\partial \rho\alpha}{\partial t} + \dfrac{\partial \rho\alpha u}{\partial x} + \dfrac{\partial \rho\alpha v}{\partial y} = 0 \\ \dfrac{\partial w}{\partial t} + \dfrac{\partial \mathrm{F}}{\partial x} + \dfrac{\partial \mathrm{G}}{\partial y} = \mathrm{S} \end{cases}$$

by

$$\begin{pmatrix} \rho\alpha \\ w \end{pmatrix}_{ij}^{n*} = \begin{pmatrix} \rho\alpha \\ w \end{pmatrix}_{ij}^{n} - \Delta t \left(\frac{f_{i+1/2} - f_{i-1/2}}{\Delta x} + \frac{g_{j+1/2} - g_{j-1/2}}{\Delta y} \right) + \Delta t\, \mathrm{S}_{ij}$$

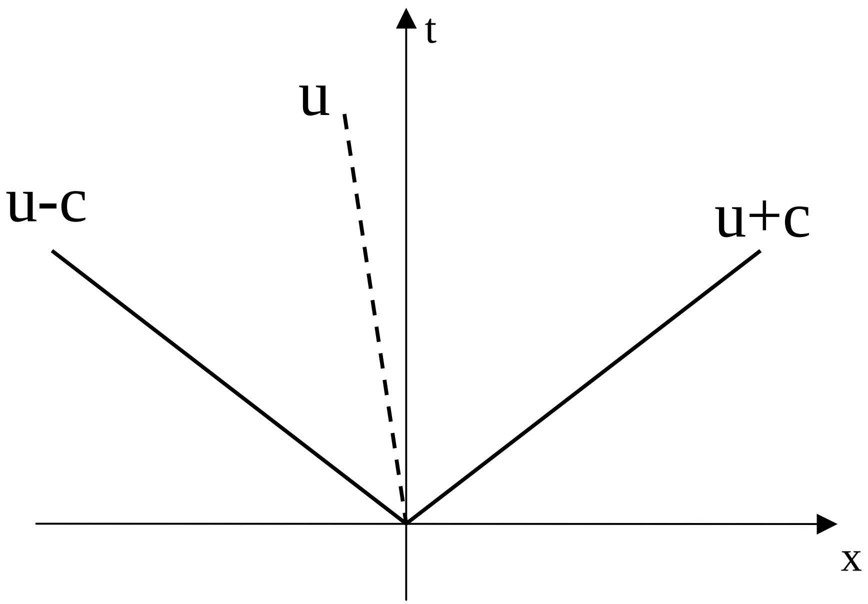
to obtain intermediate state $(\alpha, w)^{n*}$

Δt is fixed by a CFL condition

Numerical Method : hyperbolic step
numerical scheme

Numerical flux functions *f* and *g* based on exact resolution of the associated Riemann problem :

Godunov Scheme



	u-c	u	u+c
Nature	shock or rarefaction	contact discontinuity	shock or rarefaction

Numerical Method :
pressure relaxation step

Given $(\alpha, w)^{n*}$, we solve with $\varepsilon \to 0$:

$$\begin{cases} \dfrac{\partial \alpha}{\partial \tau} = \dfrac{p_g - p_\ell}{\varepsilon} \\ \dfrac{\partial w}{\partial \tau} = 0 \end{cases}$$

only α change

to obtain α^{n+1} and w^{n+1} :

$$\begin{cases} \alpha^{n+1} = \alpha^*\left(\tilde{\rho}_1^{n*}, \tilde{\rho}_2^{n*}\right) \\ w^{n+1} = w^{n*} \end{cases}$$

no numerical scheme is needed for the volume fraction with this approach

<u>Remark :</u> α^* can be explicitly computed for linearized EOS

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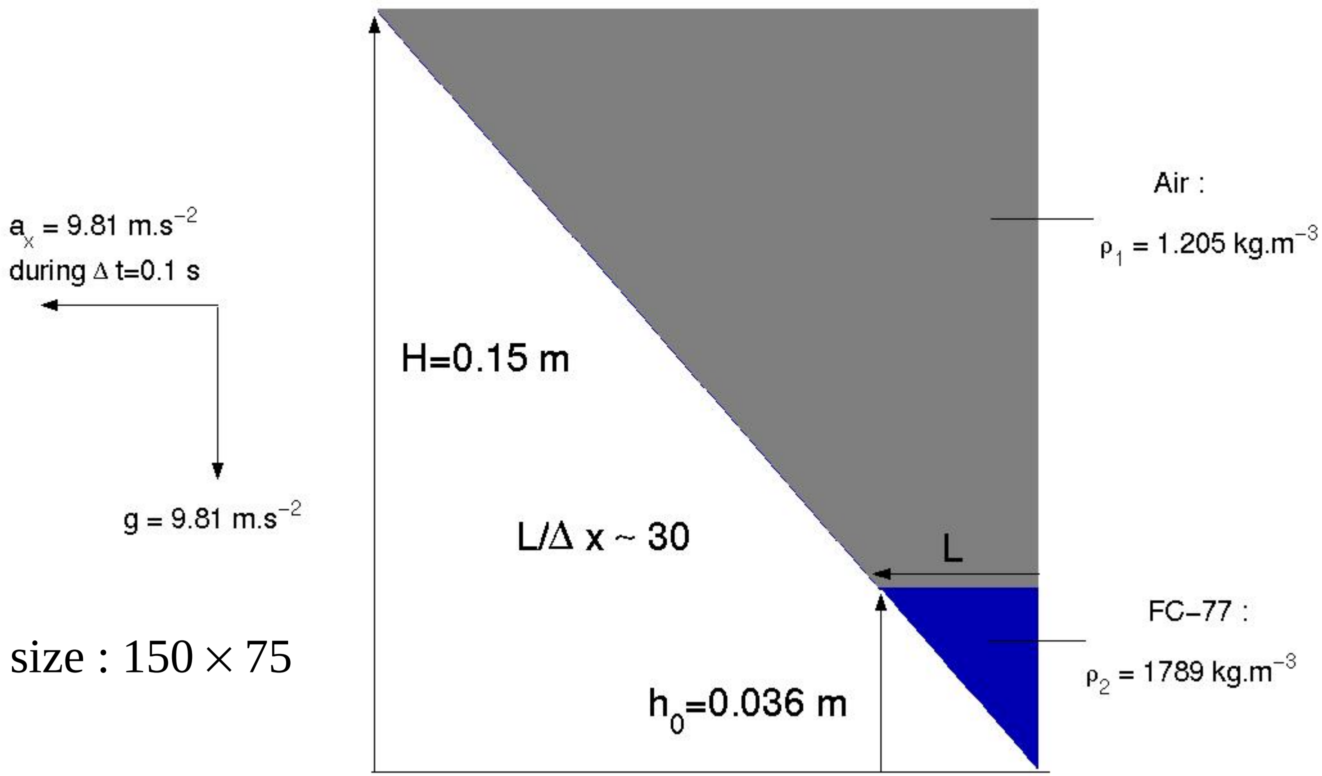
Numerical Results : Sloshing in a 2D wedge
high Froude number, high Bond number



mesh size : 150 × 75

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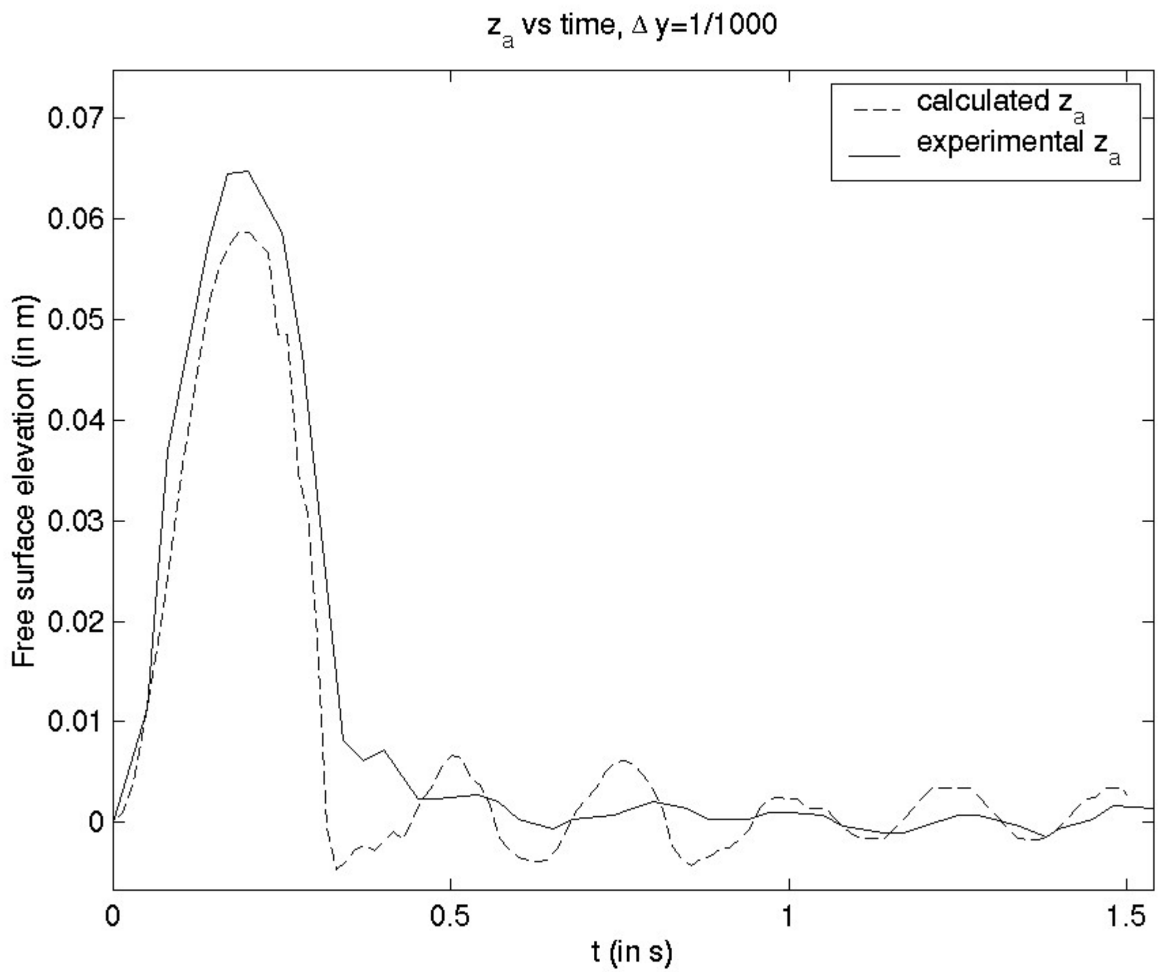
Numerical Results : Sloshing in a 2D wedge
high Froude number, high Bond number



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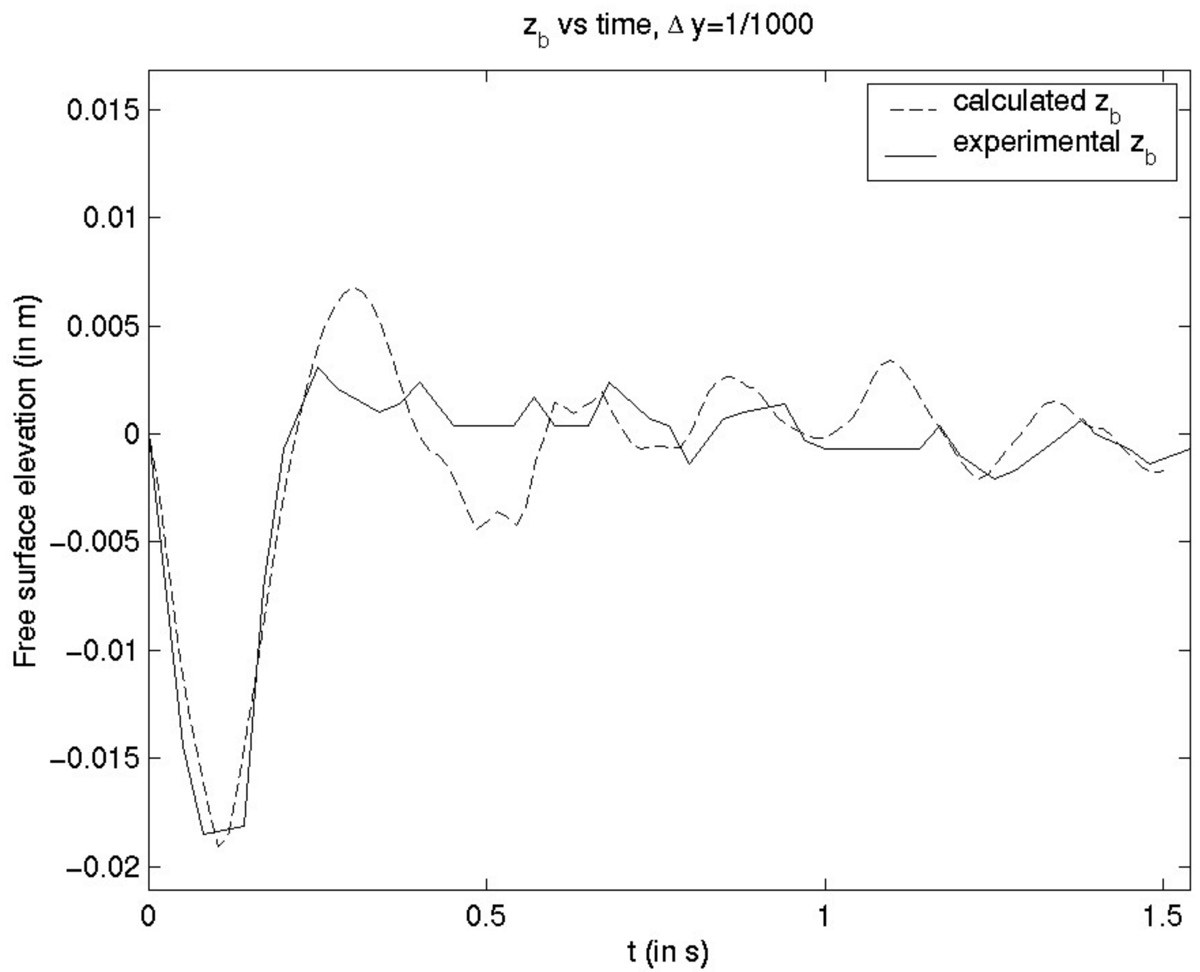

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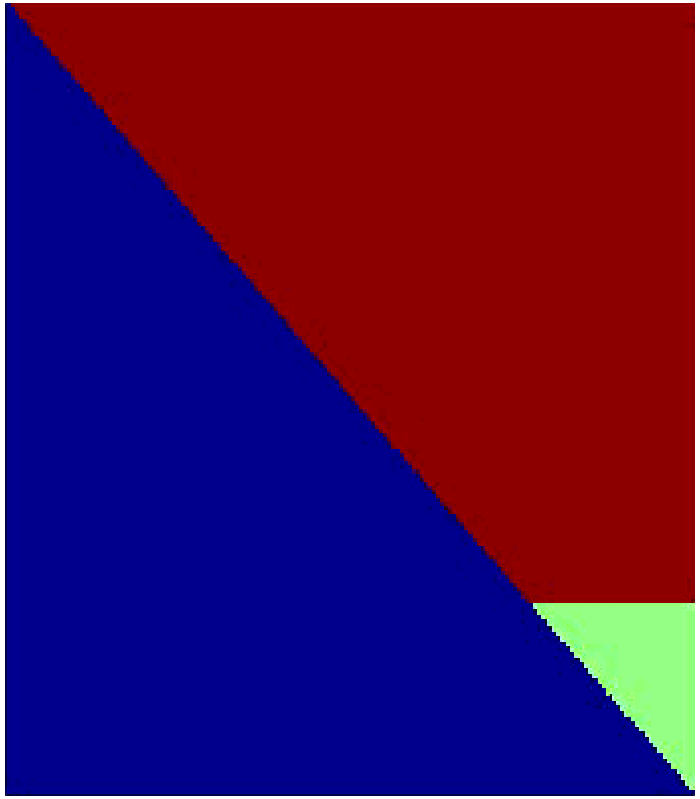

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Physically Relevant Quantities

- Total gas volume : $V_g = \int_\Omega \alpha \, dV$

- Energies : $$F_{tot} = \underbrace{\int_\Omega F \, dV}_{\substack{\text{pressure part of the free energy } (F_p) \\ + \\ \text{kinetic energy } (F_{ec})}} + \underbrace{\int_\Omega \sigma \|\nabla \alpha\| \, dV}_{\substack{\text{remaining part of} \\ \text{the free energy :} \\ \text{capillary energy } (F_{ts})}}$$

This total energy F_{tot} is decreasing for the relaxation model (up to a slight modification of the pressure closure law, current work with D. Jamet, CEA)

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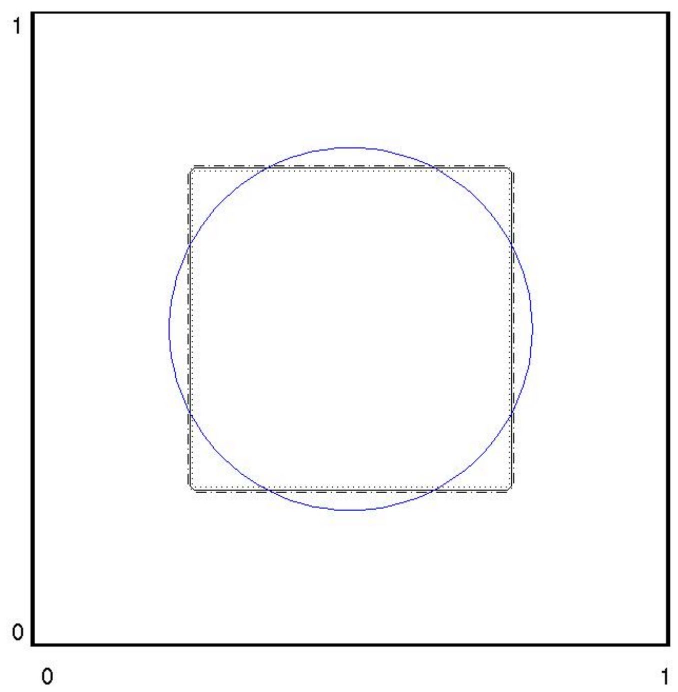
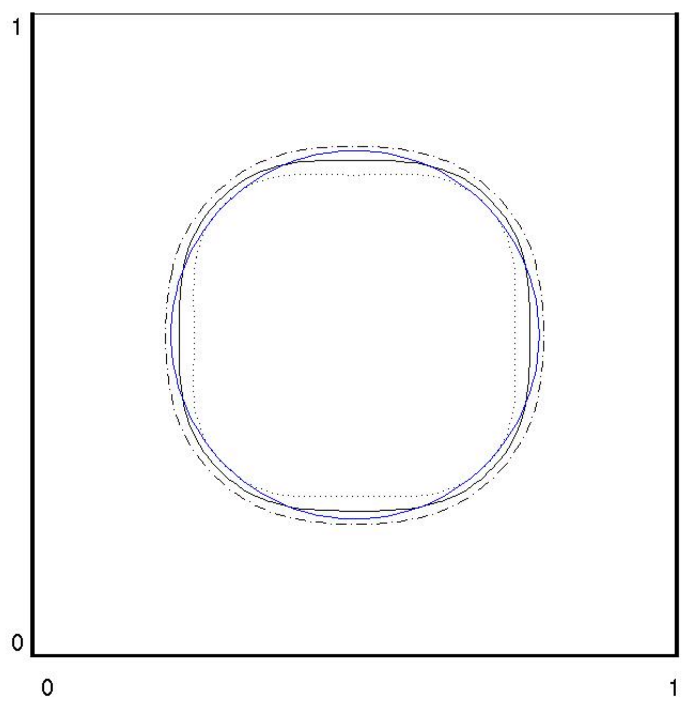
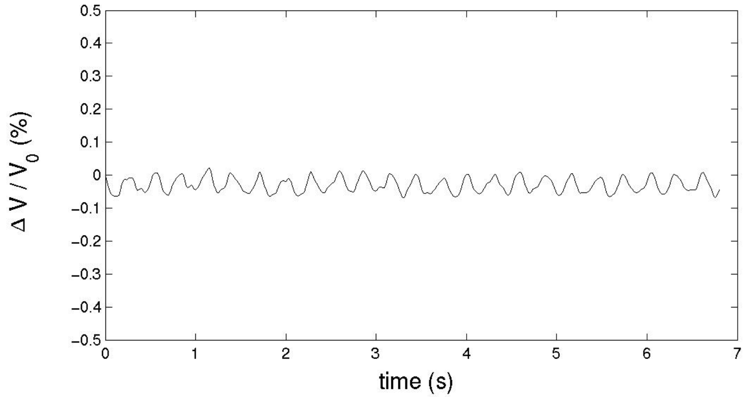
Numerical Results : square to circular bubble

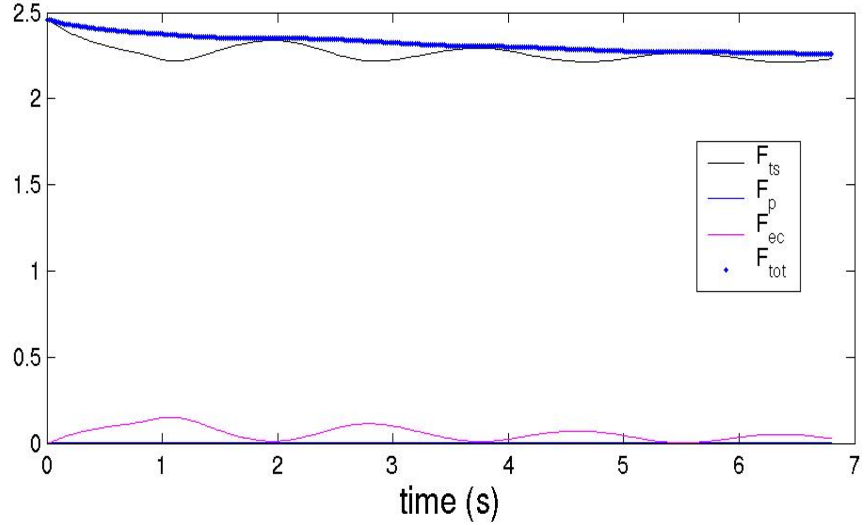




mesh size : 80×80

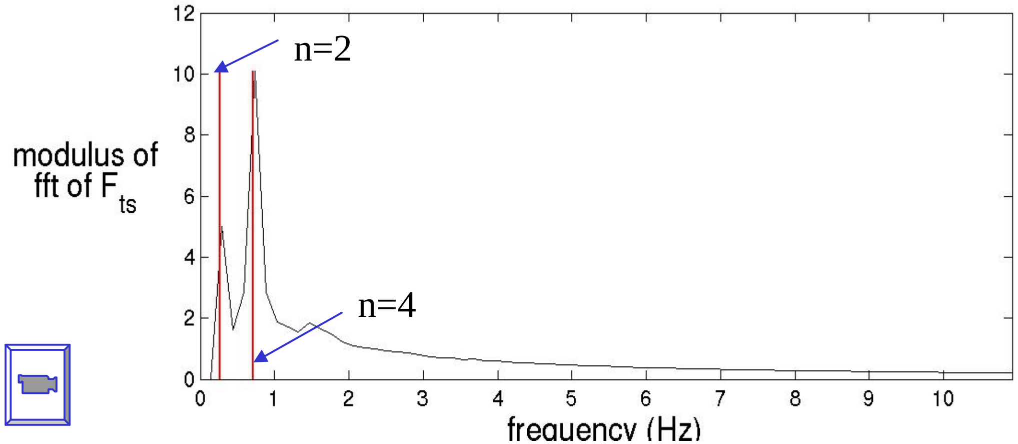
Numerical Results : square to circular bubble
energy balance





Bubble oscillations in a inviscid fluid (Lamb, 1932), deformation modes frequencies :

$$f_n = \frac{1}{\pi}\sqrt{(n+1)(n-1)(n+2)\frac{\sigma}{\rho_\ell R^3}}$$



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Conclusions and Future Developments

- Model and numerical method efficient and accurate for sloshing effects at high Bond number :
  - no numerical scheme required for the gas volume fraction
  - numerical scheme with very low diffusivity compared to similar approaches $\Rightarrow$ low damping of the interface oscillations
  - possible extension of the Riemann solver to more complex EOS (local linearization)

- Capillary forces : validation in progress
- Free energy balance with surface tension effect
- Introduction of viscous and thermal effects ($p(\rho,T)$, conduction, cavitation, evaporation)