TP Scilab N° 0

Tutorial Scilab

Note: The aim of this first workshop is to learn the software Scilab. There is no point in doing the computations by hand. If you know already Scilab (or Matlab or Octave), you can proceed directly with TP 1.

Exercice 1 Consider $u$, $v$ and $w$ the following vectors

$$u = (1, -1, 2)^T, \quad v = (10, -1, 3)^T, \quad w = (5, -1, 4)^T.$$  

1. Compute $3u$, $\|u\|_2$, $2u - v + 5w$, $\|2u - v + 5w\|_1$, $\|w - 4v\|_\infty$.

2. Find the angle formed by vectors $v$ and $w$.

Exercice 2 We note $u$ and $v$ the complex numbers

$$u = 11 - 7i, \quad v = -1 + 3i.$$  

1. Compute the complex modulus (complex norm) of $u$ and of $v$, the products $uv$ and $vu$, the real part and the imaginary part of $u^3 + v^2$.

2. We pose

$$A = \begin{pmatrix} u & v \\ \bar{u} & \bar{v} \end{pmatrix}.$$  

Compute $AA^*$ and $A^*A$.

Exercice 3 We define the vectors $u_1$, $u_2$, $u_3$ and $u_4$ de $\mathbb{R}^5$ as

$$u_1 = \begin{pmatrix} 1 \\ -3 \\ 3 \\ 5 \\ 4 \end{pmatrix}, \quad u_2 = \begin{pmatrix} 0 \\ 1 \\ 2 \\ 3 \end{pmatrix}, \quad u_3 = \begin{pmatrix} 2 \\ -5 \\ -1 \\ -6 \end{pmatrix}, \quad u_4 = \begin{pmatrix} 3 \\ 4 \\ 5 \\ -2 \\ 0 \end{pmatrix}.$$  

Let $A$ bet the matrix whose columns are formed by vectors $u_1, \ldots, u_4$. What is the rank of $A$? Same question if we replace $u_4$ by the vector $(-3, 11, 4, 13, 4)^T$. Find the kernel of $A$.

Exercice 4 We note $A$, $B$ and $C$ the following matrices

$$A = \begin{pmatrix} 1 & 3 & 2 \\ -5 & 3 & 1 \\ -10 & 0 & 3 \\ 1 & 0 & -2 \end{pmatrix}, \quad B = \begin{pmatrix} 1 & -2 & 5 \\ 6 & 1 & -1 \end{pmatrix}, \quad C = \begin{pmatrix} 10 & -5 \\ 3 & 1 \end{pmatrix}. $$
1. Compute the matrices $AB, BA$ and $AB^T$.
2. Compute the matrix $D = I_2 - BB^T$.
5. Compute the eigenvalues of matrix $E$. What is the spectral radius of $E$?
6. Find the eigenvectors of matrix $E$.

**Exercice 5** We pose

\[
A = \begin{pmatrix} 1 & -1 & 7 \\ -4 & 2 & 11 \\ 8 & 0 & 3 \end{pmatrix}, \quad B = \begin{pmatrix} 3 & -2 & -1 \\ 7 & 8 & 6 \\ 5 & 1 & 3 \end{pmatrix}.
\]

What are the following commands doing

3*A; A*B; A./B; A.^B; cos(A); exp(B); C=[A B]; D=[A,B]; E=[A;B];

**Exercice 6** For each of the following matrices

\[
A_1 = \begin{pmatrix} 1 & 2 & 3 \\ 3 & 2 & 1 \\ 4 & 2 & 1 \end{pmatrix}, \quad A_2 = \begin{pmatrix} .75 & 0 & .25 \\ 0 & 1 & 0 \\ .25 & 0 & .75 \end{pmatrix},
\]

\[
A_3 = \begin{pmatrix} .375 & 0 & -.125 \\ 0 & .5 & 0 \\ -.125 & 0 & .375 \end{pmatrix}, \quad A_4 = \begin{pmatrix} -.25 & 0 & -.75 \\ 0 & 1 & 0 \\ -.75 & 0 & -.25 \end{pmatrix},
\]

compute $A_i^n$, for $n = 1, 2, 3 \ldots$ What is the value of $\lim_{n \to \infty} A^n$?

**Exercice 7** Consider the matrices $A$ and $B$

\[
A = \begin{pmatrix} 1 & 3 & 2 \\ -5 & 5 & 1 \\ -10 & 0 & 3 \end{pmatrix}, \quad B = \begin{pmatrix} 1 & -2 & 5 & 7 \\ 6 & 1 & -1 & 3 \\ 1 & -3 & 4 & 2 \end{pmatrix}.
\]

1. We note $u$ the second column of $A$ and $v$ the last row of $B$, find the matrix $uv$.
2. We note $C$ the matrix obtained from the matrix $A$ by replacing $u$ by $-u$, find $C$.
3. We pose $C = AB$. What are the following commands doing

   x=A(3); y=B(4); x*y
   C(2:3,1:3)
   C(:)

4. What are the following commands doing

   E=A(2:3,1:3)
   find(E>0)
   if find(E>0) then s=1 end;
Exercice 8 What are the following commands doing

```matlab
for i=1:3:11, i, end;
u=[1,4,7,10];
for i=u, i, end;
v=1:3:11;
for i=v, i, end;
```

Exercice 9 We pose

\[
A = \begin{pmatrix}
1 & -1 & 7 \\
-4 & 2 & 11 \\
8 & 0 & 3
\end{pmatrix}, \quad B = \begin{pmatrix}
3 & -2 & -1 \\
-7 & 8 & 6 \\
-5 & 1 & 3
\end{pmatrix}.
\]

What are the following commands doing

```matlab
v=[1, 2, 4, 7];
A(v)=A(v) + 0.01;
B(v)=abs(B(v));
```

Exercice 10 Write a function that, given an integer \(k\) and a matrix \(X\), compute the integers \(i\) and \(j\) such that \(X(i,j) = X(k)\).

Exercice 11 Write a function that replace all coefficients \(a_{i,j}\) strictly positive of a matrix by 0 while keeping the others unchanged.

Exercice 12 1. Write a function that returns the indices \(i\) and \(j\) of the elements \(X_{i,j}\) of a matrix \(X\) of size \(n \times m\), that are larger than a given value \(a\).
2. Write a function that returns the indices \(i\) and \(j\) of the elements \(X_{i,j}\) that are included between two given values \(a\) and \(b\).

Exercice 13 The following programs must be written in a file and executed from the Scilab window.

1. Write a function that returns a real value \(a = |a_{i,j}|\) such that \(|a_{i,j}| = \max_{i,k} |a_{i,k}|\). The complex numbers \(a_{i,k}\) are the components of a matrix \(A\) of size \(m \times n\) send as input argument of the function.
2. Write a function that returns the indices \(i\) and \(j\) corresponding to the previous number \(a\).