3-D geometric modeling of a draped woven fabric

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Abstract

One of the components required for design and optimisation of fabric reinforced products is a realistic and accurate 3-D geometry model for a repeating element of the reinforcement. Such a model, which reflects the deformations that result from weaving the fibre bundles and forming (draping) the woven fabric over a product mould, is proposed in the present paper. A fibre bundle architecture, which exhibits undulation and variable cross-section dimensions, is introduced to this effect. Every bundle is described by its in-plane centreline path, its double curved horizontal midplane and the thickness distribution of the cross-sections. These parameters are, in turn, defined by invariant shape functions and variable fibre bundle dimensions. The selected geometry description enables straightforward determination of individual fibre paths. In order to verify the model experimentally, cross-sections are cut out from undeformed laminates with plain-weave reinforcements, and laminates of which the plain-weaves have been subjected to stretching or shear deformations during draping. The laminate cross-sections are made along and perpendicular to the mean directions of the impregnated fibre bundles (yarns). All yarns exhibit out-of-plane undulation, and curvature and twist of the midplane. Correlation between experiment and the proposed modeling scheme is good. Draping results in significant fibre reorientations and variations between the individual fibre paths, which are not reflected by existing modeling schemes. These geometry deviations may significantly affect the stress distribution, and should be taken into account in order to predict material properties accurately. © 2001 Elsevier Science Ltd. All rights reserved.

Keywords: 3-D geometry model; Woven fabric; Draping; Fibre reorientations; Plain-weave

1. Introduction

Continuous fibre reinforced composites offer high potential for use in aerospace and automotive industry thanks to the endless possibilities to tailor their stiffness and strength properties. Fabric reinforcements, that consist of interlaced continuous fibre bundles, have the advantages of easy handling and formability, which makes them suitable for use in complex shaped structural parts. For design of these parts, a combination of three tools will be of considerable assistance [2], namely: (i) forming or draping simulations, (ii) constitutive models for the effective material properties, and (iii) finite element (FE) simulation techniques. Fig. 1 shows schematically the interaction between these tools within the design process.

Draping simulations (Fig. 1(a)) predict the in-plane fibre bundle reorientations that result from forming the fabric onto the product mould. As an example, a kinematic drape simulation of a closed semi-cylinder is depicted in the figure. For such draping simulation techniques, final fibre bundle orientations are generally obtained as a function of production variables, the initial fabric positioning and the product shape. This is achieved by draping a so-called fisherman’s net over the product shape. The fisherman’s net consists of inextensible line segments, representing the in-plane fibre bundle directions, which are connected at their crossover points by pivots.

Constitutive models (Fig. 1(b)) are required in order to assign effective material properties to every fisherman’s net cell, as a function of the fibre reorientations which are induced by the forming process. To this effect, a repeating volume element (RVE) of the reinforced lamina or laminate is regarded, which is acceptable if the loading condition and the geometry of the reinforcements are nearly constant throughout the area covered by the fisherman’s net cell. The RVE consists of the deformed, impregnated fibre bundles and the surrounding pure resin regions, and is generally assumed to be flat, i.e., bending deformations of the lamina(te) are neglected. Accurate prediction of the RVE effective material properties requires a realistic
geometry model of the deformed reinforcement fabric. In a sense, the geometry model can be constructed by extending the fisherman’s net geometry, which merely gives information on the in-plane reorientation of the fibre bundles, to a certain required level of detail [3], as schematised in Fig. 1(b).

The effective material properties are used as input for a macro-mechanical analysis of the structure (Fig. 1(c)) using, e.g., FE simulation techniques. It should be emphasised that every single integration point will in principle be assigned different material properties, depending on draping induced fibre reorientations. As the requirements for the FE mesh generally differ from those for the draping simulation and material properties calculation [2], the mesh is superimposed over the extended fisherman’s net, as depicted in Fig. 1(c). In addition to a single analysis step, the three modules described above can be incorporated in a design tool for shape optimisation, see [4].

Two approaches were recently developed by the authors for calculation of the effective elastic stiffness properties of a deformed plain-weave RVE [3,5,6]. The fisherman’s net was extended (Fig. 1(b)) in order to account for out-of-plane undulation of the fibre bundles. To this effect, the inextensible line segments, that reflect in-plane fibre bundle orientations for the purpose of kinematic drape simulations, were substituted by inextensible undulated fibre bundle centrelines [3]. All individual fibre paths were assumed to be parallel to this centreline path, which implies constant fibre bundle cross-sections [7]. Not only the undulation induced by the weaving process, but also changes in undulation caused by fabric shear deformations and fabric stretching deformations, which may occur during draping, were reflected by the model. Stiffness of the impregnated fibre bundles or yarns was reduced for the effect of undulation with a homogenisation procedure assuming either constant strain (iso-strain) or constant stress (iso-stress) along the schematised yarn centreline [3]. Subsequently, the two yarn directions and the pure resin material were replaced by three substitute layers. Stiffness of this virtual laminate, which replaced the fabric reinforced RVE, was calculated with classical laminate theory (CLT). The approaches were denoted the CLT iso-strain and CLT iso-stress model for yarn properties reduced with an iso-strain and an iso-stress assumption, respectively. In-plane elastic stiffness predictions were compared with tensile test results for sheared and stretched carbon and glass fibre reinforced laminates [6]. Good correspondence between theory and experiment was found.

For accurate prediction of non-linear material behaviour, damage and failure, however, the CLT models are not adequate due to the highly simplified assumed stress and strain distributions. In these cases, method of cells based approaches [8,9] and FE [10–14] approaches, that rely on a 3-D geometric description of the reinforcement, appear to be especially promising. These micromechanics approaches give a more realistic prediction for the stress and strain distributions inside the RVE. However, in spite of their high resolution, the underlying 3-D geometry models currently available in the literature incorporate a fixed and idealised geometry. For woven fabrics, fibre bundle geometry is schematised according to the interlacing pattern by orthogonally orientated, undulated centrelines. The draping induced in-plane fibre bundle reorientations, that were included in the CLT models described above, are not reflected. In order to obtain the 3-D fibre bundle geometry, generally a constant cross-section is swept along the centrelines. Yet, in an actual component, the fibre bundle cross-section will be subjected to various draping induced deformations. This will lead to variations in orientation of the individual fibres [7].
Therefore, a 3-D geometric modeling scheme is introduced in the present work, which reflects:
- In-plane fibre bundle reorientations after draping of the fabric.
- Out-of-plane undulation. In addition, possible in-plane undulation of the fibre bundles can be reflected with the modeling scheme.
- Variable cross-section geometry along the fibre bundle centreline.
- Changes in fibre bundle cross-section dimensions and undulation caused by the forming process.

The model can be regarded as a 3-D extension of the fisherman’s net model depicted in Fig. 1(b), and is, in the first place, intended as a basis for micromechanics calculations of the effective RVE material properties. The geometry model is presented here for deformed plain-weave reinforcements. However, a number of considerations concerning an extension to other weave types, such as twill-weaves and satin-weaves, are given as well. Geometrically balanced as well as geometrically non-balanced plain-weaves, i.e., plain-weaves with different dimensions of the warp and fill fibre bundles, are regarded. Furthermore, an experimental verification is presented by comparing the 3-D model to laminate cross-section photomicrographs.

The present paper is organised as follows. The proposed fibre bundle architecture, and a classification of deformed fibre bundle configurations are presented in Sections 2.1 and 2.2, respectively. The methodology applied for 3-D modeling of a deformed, geometrically non-balanced plain-weave is the subject of Section 3.1, while a simplified configuration is treated in Section 3.2. Examples are presented in Section 4. Plain-weaves are modeled in undeformed, sheared and stretched configurations and compared to laminate cross-section photomicrographs. Finally, conclusions and an outlook are given in Section 5.

2. Deformed fibre bundle geometry

2.1. Fibre bundle architecture

The proposed geometry model for a fibre bundle, which reflects geometry deviations caused by, e.g., weaving and draping the fabric, is depicted in its local cartesian coordinate system in Fig. 2. Firstly, the centreline is regarded, which is depicted in Fig. 2(a). A single wavelength (l) will be modeled, which may exhibit both in-plane and out-of-plane undulation. However, in a macroscopic sense (e.g. on the length scale of a fisherman’s net cell) the fibre bundle path is assumed to be straight. The x'-axis coincides with the macroscopic mean direction. The (x',y')-plane lies parallel to the fabric midplane, and z' is the out-of-plane direction. The origin of the x'-axis is placed at a crossover point of two fibre bundle centrelines. At this crossover point, the (x',z')-plane is assumed to be a plane of symmetry for the centreline. It should be noted that additional crossover points will be present along the wavelength, its number depending on the weave type. A normalised coordinate along the mean fibre bundle direction is introduced, which is defined by

$$\zeta' = \frac{x'}{l}. \quad (1)$$

In-plane and out-of-plane undulation of the fibre bundle centreline are described with the coordinates y_c'(\zeta') and z_c'(\zeta'), respectively, which are defined as

$$y_c' = B_c Y, \quad z_c' = A_c \Phi \quad (2)$$

![Fig. 2. Schematisation of a fibre bundle which reflects processing induced geometry deviations. (a) wavelength and local coordinate system, (b) fibre bundle cross-section in a non-woven state (top) and with processing induced geometry deviations (bottom), (c) orientation of an individual fibre.](image-url)
with \( B \) and \( A \) the undulation amplitudes, see Fig. 2(a), and \( Y(\xi') \) and \( \Phi(\xi') \) pre-defined shape functions.

On a higher level of geometric detail, the fibre bundles consist of thousands of individual fibres. In the literature [10–17], 3-D fabric geometry is frequently defined by sweeping a constant cross-section along the centreline, which implies parallel fibre paths [7]. Rather than assuming all fibre paths to be parallel to the centreline, however, individual fibre paths will be reflected with the present modeling scheme. To this effect, a variable fibre bundle cross-section is introduced, which is depicted for a non-woven state and a deformed state in Fig. 2(b). Fibre bundle cross-sections are defined parallel to the \((y', z')\)-plane, as will be treated further in the following. For the non-woven configuration, all fibres are parallel and straight. After weaving of the fibre bundles and handling and processing of the fabric this is not anymore the case. The deformed fibre bundle is described with variable cross-section dimensions \( a(\xi') \) and thickness \( t(\xi') \), cross-section thickness distribution \( h(\xi', \eta') \) and a double curved horizontal midplane, defined by the out-of-plane coordinate \( z_m(\xi', \eta') \).

In order to define the relative location of an individual fibre in a cross-section, two additional normalised coordinates are introduced. The normalised transverse location of a fibre slice with respect to the centreline \( (\eta') \) and the normalised out-of-plane location of an individual fibre with respect to its horizontal midplane \( (\zeta') \) are defined as

\[
\eta' = \frac{y' - y'_0}{a}, \quad \zeta' = \frac{z' - z'_m}{h}.
\]  

Thus, every fibre bundle cross-section is defined on the intervals \( \eta' \in [-\frac{1}{2}, \frac{1}{2}] \) and \( \zeta' \in [-\frac{1}{2}, \frac{1}{2}] \).

For all possible configurations, it is assumed that:

- Fibre packing is constant for every individual fibre bundle cross-section, i.e. the fibres are distributed evenly, although fibre packing may vary as a function of \( \zeta' \).
- The relative positions of a fibre with respect to the centreline and with respect to the horizontal midplane are constant. This also implies that fibres do not interlace.

The latter assumption is expressed as

\[
\eta' = \text{constant}, \quad \zeta' = \text{constant}
\]  

for an individual fibre.

With the assumptions above, the cross-section thickness distribution can be described as

\[
h = t\Psi
\]  

with \( \Psi(\eta') \) an invariant shape function, i.e. independent of the cross-section under consideration. Furthermore, the shape function is assumed to be identical for all fibre bundle configurations, which implies that weaving and processing of the fabric do not affect the fibre distribution.

The horizontal midplane coordinate will be determined for every fibre bundle slice \( (dy' = dy'/a) \) individually, see Fig. 2(b). Corresponding to the centreline path treated above, the out-of-plane slice path is described as a function of its amplitude \( (A(\eta')) \) and a shape function \( (\Phi(\xi')) \) as

\[
z_m' = A\Phi.
\]  

As can be derived from (3) and Fig. 2(a) and (b), \( z_m'(\xi', 0) = z_m'(\xi') \) and \( A(0) = A_c \). It is assumed that the shape function is identical for all fibre bundle slices. However, fibre slices may exhibit variable paths defined by their respective amplitudes, and a phase shift between individual slices may be present for sheared configurations (see Section 3). Furthermore, individual fibres present in a slice will exhibit variable out-of-plane orientations when the local thickness varies along the mean fibre bundle direction. Appropriate shape functions \( (\Phi(\xi')) \) will be selected according to the weave type, see Section 3.

Output of the geometric model, required for determination of material properties, will be the fibre distribution and the orientation of the fibres. For constant fibre packing, the fibre distribution follows directly from \( h(\xi', \eta') \) and the packing ratio. Local orientation of an individual fibre with respect to the mean fibre bundle axis is defined by the out-of-plane crimp angle \( \omega(\xi', \eta') \), i.e. the angle between the fibre projection on the \((x', z')\)-plane and the \(x'\)-axis, and the in-plane crimp angle \( \chi(\xi', \eta') \), i.e. the angle between the fibre projection on the \((x', y')\)-plane and the \(x'\)-axis. These angles are depicted in Fig. 2(c). For fabrics applied in structural parts, it is assumed that both crimp angles are moderate along the undulated path for every individual fibre in the bundle. This assumption is expressed as

\[
\omega^2 \ll 1, \quad \chi^2 \ll 1.
\]  

An individual fibre is assumed to act as an inextensible linear elastic beam. This is acceptable for most (glass, carbon) reinforcement fibres which exhibit high longitudinal stiffness. The assumption implies that the cross-section definition adopted in the present work, i.e. normal to the mean fibre direction, is physically not correct. A more valid geometry description would probably be obtained by introducing a curvilinear coordinate system with curved axes along the centreline, the midplane and the slice. The resulting cross-section description would, however, likely lead to very complex geometric relations, especially when contact conditions between fibre bundles need to be satisfied, as is the case in the present work. Regarding the Taylor expansions of the fibre orientations in combination with (7), it can be derived that \( \cos \omega \approx 1 \) and \( \cos \chi \approx 1 \). It thus follows that, for moderate crimp, the cross-section definition
normal to the mean fibre bundle direction, as advocated here, is a valid approximation to a cross-section definition normal to the local fibre direction.

The fibre orientation relative to the mean fibre bundle direction can be derived from
\[ \tan \omega = -\frac{\partial z'}{\partial x'}, \quad \tan \chi = \frac{\partial y'}{\partial x'}. \] (8)

Combining (1), (2), (4) and (6)-(8) gives
\[ \omega = -\frac{1}{I} \left[ \zeta \frac{dt}{ds} + A \frac{d\Phi}{ds} \right], \quad \chi = \frac{1}{I} \left[ \eta' \frac{d\alpha}{ds} + B \frac{d\gamma}{ds} \right], \] (9)

where (7) is applied in combination with Taylor expansions, which give \( \tan \omega \approx \omega \) and \( \tan \chi \approx \chi \).

### 2.2. Deformed fibre bundle configurations

Fig. 3 shows 3-D plots for the fibre bundle configurations that can be described with the proposed architecture. The paths of the outer fibres, which compose the fibre bundle 'surface' depicted in the figure, are defined by \( \zeta' = \pm 1/2 \), as can easily be derived from (3) and Fig. 2(b). Geometric parameters that vary as a function of the normalised mean fibre bundle direction \( \zeta' \) and/or the normalised transverse coordinate \( \eta' \) are depicted for every configuration. These parameters have been defined in Fig. 2(a) and (b).

A fibre bundle which exhibits out-of-plane undulation is schematised in Fig. 3(a). Out-of-plane undulation is caused by weaving the fibre bundles and affected by draping the fabric [3]. For this configuration, fibre bundle cross-section geometry is constant. As a consequence, all individual fibre paths are parallel and the deformation is fully described by the out-of-plane centreline coordinate \( z_m'(\xi') \).

Other deformed configurations exhibit twisted (Fig. 3(b)) and curved (Fig. 3(c)) midplanes. For both configurations, the resulting horizontal midplane will be double curved. As will be shown in Section 4, a sheared plain-weave may exhibit fibre bundles with significant twisting deformations, whereas fibre bundles in a stretched plain-weave may exhibit significant cross-section midplane curvature.

Additional geometry deviations depicted in Fig. 3(d)–(f) are not present in a significant manner in the plain-weaves that will be considered in the following, see Section 4. These configurations are, however, observed for other weave types. This will be shown by example of a sheared satin-weave. A top-view of an impregnated lamina, and a laminate cross-section photomicrograph along the \( x' \)-direction are depicted in Fig. 4(a) and (b), respectively. A 7-harness balanced satin-weave is considered. The notation 7-harness implies that one interlacing is followed by an array of seven successive over- or undercrossed fibre bundles. Such an array is referred to as a float. Long floats allow for a number of additional deformations to take place. Firstly, the floats will locally exhibit higher width and probably lower thickness than the interlacing region. This difference between the floats and the interlacings can be described with two deformed configurations, i.e. fibre bundle compaction (Fig. 3(d)) and pinching (Fig. 3(e)). For a compacted fibre bundle, both cross-section width and thickness decrease along the mean fibre bundle direction, while the aspect ratio \( (a/t) \) remains constant. This leads to a locally increased fibre packing which may

![Fig. 3. Fibre bundle configurations; (a) undulated (out-of-plane), (b) twisted, (c) with curved horizontal midplane, (d) compacted, (e) pinched, (f) undulated (in-plane).](image-url)
significantly affect strength properties. For the pinched fibre bundle, cross-section area and fibre packing remain constant, while width decreases and thickness increases accordingly in the mean fibre bundle direction. The described phenomena were observed on satin-weaves by Gao et al. [18]. From the top-view of Fig. 4(a), it is seen that pinching and/or compaction take place at the interlacings of the sheared 7-harness satin-weave regarded here as well.

Secondly, long floats may allow for in-plane undulation of the fibre bundles. Minor in-plane undulation is observed from the photomicrograph of Fig. 4(a), but this deformation may become more pronounced for fabrics with higher shear deformations. In-plane undulation is schematised in Fig. 3(f). Equivalent to the out-of-plane undulated configuration, cross-section geometry remains constant and all fibres run parallel.

i.e., a plain-weave with different geometric properties in warp and fill fibre bundle directions [5]. As has been mentioned in Section 1, it is expected that the approach can relatively easy be expanded to other weave types.

Firstly, a weave repeat is defined for the deformed woven fabric. Required input is schematised in Fig. 5. Draping simulations (Fig. 5(a)) provide the inter-yarn angle ($\theta$) and the local cartesian fibre bundle coordinate systems ($x', y', z'$) with respect to a global cartesian coordinate system ($x, y, z$). Warp and fill fibre bundle directions are denoted with subscripts $w$ and $f$, respectively. For relations that apply to both fibre bundle directions, these subscripts will be omitted. Out-of-plane bending of the fabric is neglected on the length scale of a weave repeat. The weave diagram, depicted in

3. 3-D extended fisherman\'s net model for a weave repeat

3.1. Geometrically non-balanced plain-weave

The proposed fibre bundle architecture is next applied to model a woven fabric for use as an interface between a kinematic draping simulation and a material model. The procedure which is followed to this effect is illustrated for a geometrically non-balanced plain-weave,
Fig. 5(b) for a plain-weave, presents the number of fibre bundles that compose the weave repeat and their interlacing pattern. Further input required at this stage are the fibre bundle cross-section dimensions \((a, i, t)\) and the wavelengths \((l)\).

With the input above, a parallelepiped shaped weave repeat is constructed, as schematised in Fig. 6(a). This weave repeat will serve as the basis for the 3-D extended fisherman’s net model. The first warp and fill fibre bundle start at the left bottom of the weave repeat. For any weave type, geometry of a consecutive warp or fill fibre bundle can be obtained by translating geometry of the first fibre bundle along \(x'\)-direction. For a plain-weave, this translation amounts to half a wavelength, as can be derived from Fig. 6(a). Therefore, only the first warp and fill fibre bundle need to be modeled. As has been proposed in Section 2.2, the cross-section dimensions are assumed constant for every fibre bundle in a plain-weave:

\[
a = \text{constant}, \quad t = \text{constant}.
\]

Furthermore, in-plane undulation is neglected for a plain-weave, which gives (Fig. 2(a))

\[
B_c = 0.
\]

A useful geometric parameter, which can be derived from Fig. 6, is the in-plane dimension of the crossover region \((a_{b_w})\). This parameter is depicted in Fig. 6(b) and (c), showing side views along the \(x'_w\)-axis and the \(x'_f\)-axis, respectively. The parameter can be calculated as the projection of the width of a crossed fibre bundle on the respective \(x'_f\)-axis. After normalisation with respect to the wavelength, the in-plane dimensions of the crossover region along the warp and fill directions follow from

\[
a_{b_w}^* = \frac{1}{l_w} a_w, \quad \tilde{a}_{b_w}^* = \frac{1}{l_f} a_w,
\]

respectively. Superscript * denotes normalisation of a geometric parameter with respect to the corresponding wavelength.

For plain-weaves, in-plane contact violations between parallel fibre bundle surfaces are prohibited. For the normalised in-plane dimensions of the crossover region, this implies that

\[
a_{b_w}^* \leq \frac{1}{2},
\]

as will be confirmed by observations in Section 4.

For generation of the 3-D fibre bundle geometry, firstly the thickness distribution \((h(\eta'))\) is selected. As treated in Section 2.1, the thickness distribution is described by an invariant shape function \((\Psi(\eta'))\), and the fibre bundle cross-section dimensions \((a, i, t)\), which are constant for the plain-weaves considered here. The shape function is defined separately for the intervals \(\eta' \in [-\frac{1}{2}, 0]\) and \(\eta' \in [0, \frac{1}{2}]\). A cubic polynomial description is adopted for both intervals. Cross-sections are observed to be symmetric with respect to the centreline and to exhibit a nearly lenticular shape (Section 4). A corresponding cubic polynomial shape function is obtained with the following boundary conditions for \(\eta' \in [-\frac{1}{2}, 0]\):

\[
\begin{align*}
\Psi_{|\eta'=-\frac{1}{2}} &= 1, & \quad \frac{d\Psi}{d\eta'}\bigg{|}_{\eta'=-\frac{1}{2}} &= 0, \\
\Psi_{|\eta'=-\frac{1}{2}} &= 0, & \quad \frac{d^2\Psi}{d\eta'^2}\bigg{|}_{\eta'=-\frac{1}{2}} &= 0.
\end{align*}
\]
and, for $\eta' \in [-1/2, 1/2]$:

$$
\Psi|_{\eta'=0} = 1, \quad \frac{d\Psi}{d\eta'}|_{\eta'=0} = 0, \quad \Psi|_{\eta'=-1/2} = 0, \quad \frac{d^2\Psi}{d\eta'^2}|_{\eta'=-1/2} = 0. 
$$

(15)

Further motivation for the selected boundary conditions will be presented in the following. Applying boundary conditions (14) and (15) in order to define the cubic polynomial shape functions leads to the result

$$
h = t\Psi = t \left[ 1 - 6\eta'^2 + 4|\eta'|^3 \right], \quad \eta' \in [-1/2, 1/2].
$$

(16)

Notice that, by virtue of the adopted boundary conditions, the thickness distribution is continuous and smooth on the entire interval $\eta' \in [-1/2, 1/2]$. As has been mentioned in Section 2.1, the fibre distribution, required for calculation of material properties, follows directly from (16).

Subsequently, the horizontal midplane geometry is established. This is achieved by regarding fibre bundle slices ($d\eta' = dy/a$) individually, see Fig. 6(a). Local normalised slice coordinates parallel to the mean fibre bundle directions ($\zeta'_s$) are introduced in Fig. 6(b) and (c). These coordinates reflect the phase shifts of the slices under consideration with respect to the centrelines. The origins of the $\zeta'_s$ coordinates are placed at the centrelines of crossed fibre bundles of the other direction, and the coordinate is defined by

$$
\zeta'_s = \zeta' + k_1 \frac{a\eta'}{t \tan \theta}.
$$

(17)

The factor $k_1$ follows for the chosen definition of the inter-yarn angle from

$$
k_{wy} = -1; \quad k_{wy} = 1.
$$

(18)

Notice, however, that both configurations with $\theta \leq \pi/2$ and with $\theta \geq \pi/2$ can be described with the present architecture.

As has been treated in Section 2.1, the midplane of the slice is defined by an invariant shape function ($\Phi (\zeta'_s)$) and its amplitude ($A(\eta')$), depicted in Fig. 6. By virtue of the periodicity of the undulation, the entire fibre bundle geometry description can be related to the interval $\zeta'_s \in [-1/2, 1/2]$, which will be considered in the following.

The shape function is defined separately for the intervals $\zeta'_s \in [-1/2, 0]$ and $\zeta'_s \in [0, 1/2]$. Again, cubic polynomial descriptions are selected. Four required boundary conditions are formulated for every interval, based on observations of actual fibre bundle geometry. For $\zeta'_s \in [-1/2, 0]$:

$$
\Phi|_{\zeta'_s=0} = k_2, \quad \frac{d\Phi}{d\zeta'}|_{\zeta'_s=0} = 0, \quad \Phi|_{\zeta'_s=-1/2} = -k_2, \quad \frac{d\Phi}{d\zeta'}|_{\zeta'_s=-1/2} = 0.
$$

(19)

and, for $\zeta'_s \in [0, 1/2]$:

$$
\Phi|_{\zeta'_s=0} = k_2, \quad \frac{d\Phi}{d\zeta'}|_{\zeta'_s=0} = 0, \quad \Phi|_{\zeta'_s=1/2} = -k_2, \quad \frac{d\Phi}{d\zeta'}|_{\zeta'_s=1/2} = 0.
$$

(20)

The factor $k_2$ follows from the chosen definition for the weave repeat, and is defined for the warp and fill fibre bundles by

$$
k_{w} = 1, \quad k_{w} = -1.
$$

(21)

Applying (17) and the boundary conditions (19) and (20) in order to define the cubic polynomial shape functions leads to

$$
\Phi = k_2 \left[ 1 - 24\zeta'^2 + 32|\zeta'|^3 \right], \quad \zeta' \in [-1/2, 1/2].
$$

(22)

Again, the resulting shape function is continuous and smooth along the entire fibre bundle slice by virtue of the selected boundary conditions.

Apart from the shape functions ($\Phi (\zeta'_s)$), the amplitudes ($A(\eta')$) are required for definition of the slice midplanes. The amplitudes will be derived by assuming contact between the fibre bundles at their centrelines. Along the fill centreline ($\eta'_f = 0$), these contact conditions are formulated as

$$
z'_m + \frac{1}{2}k_3 t_f = z'_m - \frac{1}{2}k_3 h_f,
$$

(23)

whereas along the warp centreline ($\eta'_w = 0$), the contact conditions are given by

$$
z'_m - \frac{1}{2}k_3 t_w = z'_m + \frac{1}{2}k_3 h_f.
$$

(24)

The factors $k_3$ follow from the definition of the weave repeat as

$$
k_3 = 1 \quad \text{for} \quad \zeta'_s \in [-\frac{1}{2}, \frac{1}{2}a'_w],
$$

$$
k_3 = -1 \quad \text{for} \quad \zeta'_s \in [-\frac{1}{2}a'_w, \frac{1}{2}a'_w], \quad \zeta'_s \in [-\frac{1}{2}a'_w, 0],
$$

(25)

For the examples of Section 4, the normalised warp coordinates will be adopted as the global coordinate system. Coordinate transformations are thus merely required for the normalised fill coordinates and amount to

$$
\eta'_f = \frac{1}{a'_w} \left[ l_w \zeta'_w \cos \theta + a_w \eta'_w \sin \theta \right],
$$

$$
\eta'_w = \frac{1}{a'_w} \left[ -l_w \zeta'_w \sin \theta + a_w \eta'_w \cos \theta \right],
$$

(26)

as can be derived from (1), (3) and Fig. 6(a).

In order to fix their out-of-plane positions, a single value for the amplitude of either one of the two fibre bundles is required as an input parameter. In the experiments of Section 4, the amplitude of the fill centreline ($A_w$) will be measured to this effect. The amplitude of the warp centreline ($A_v$) then follows directly from
the contact condition at the crossover point \((\xi', \eta') = (0, 0)\) as
\[
A_{c_1} = \frac{1}{2}(t_c + t) - A_{c_1}.
\]  
(27)

From the thickness distributions, defined by (16), the midplane shape functions, defined by (21) and (22), contact conditions (23) and (24), coordinate transformations (17) and (26), and the relation between the centreline amplitudes (27), the following closed form solutions are obtained for the warp slice amplitudes as a function of geometric input parameters:
\[
A_w = A_c + 3 \left[ 8A_c a_{b_1}^2 - t_w \right] \eta_w^2 - 2 \left[ 16A_c a_{b_1}^2 - t_w \right] \eta_w^3,
\]
\[
\eta'_w = \left[ \frac{1}{2}, 1 \right].
\]  
(28)

The fill slice amplitudes are obtained by switching subscripts \(w\) and \(f\) in (28). Normalised crossover region dimensions \(a_{b_1}^2\) are given as a function of geometric input parameters in (12).

After combination of the midplane definition (6), the midplane shape functions (22), and the amplitudes (28), closed form expressions will result for the horizontal midplanes as a function of geometric input parameters.

Fibre orientations will be required when combining the schematised geometry with a material model. In-plane undulation has been assumed negligible and in-plane fibre orientations comply with the respective \(x^\prime\)-axes. This follows from combination of (9)–(11) as well. Out-of-plane fibre orientations are calculated from (1), (3), (9), (10), (21), (22) and (28) as
\[
\omega = -k_e \frac{A}{I} \left[ -48 c_{s_1}' + 96 \text{sign}(c_{s_1}') c_{s_2}' \right]
\]  
(29)

with normalised slice coordinates \((c_{s_1}'(\xi', \eta')), (c_{s_2}')\) from (17) and amplitudes \((A(\eta'))\) given by (28).

For constant fibre bundle thickness \((t)\), as assumed here, the angle \(\omega\) is constant for all fibres in a slice, as can be deduced from (9). The required geometric input parameters \((a, A_c, l, \omega)\) will be measured directly from laminate cross-section photomicrographs in Section 4 for a comparison with the proposed 3-D model. Obviously, measuring fibre bundle dimensions for every deformed configuration present in a product would be a very tedious if not impossible task. However, a few schematisations are available in the literature [5,19] that predict fibre bundle dimensions as a function of their values for an undeformed configuration and the inflicted deformation. When applying these schematisations, it should be taken into account whether the fabric was draped in a dry state [19], as would be the case for, e.g., the Resin Transfer Moulding (RTM) process, or impregnated with resin during forming, as would be the case for, e.g., rapid press-forming techniques for thermoplastic composites [5].

In addition to the in-plane contact conditions (13), no contact violations between fibre bundles are allowed in the out-of-plane direction. For the (representative) crossover region of the first warp and fill fibre bundles, these out-of-plane contact conditions can be written as
\[
\frac{z_{m_w}}{2} - \frac{1}{2} t_m = \frac{1}{2} t_f, \quad \xi_{s_1}' \in \left[ -\frac{1}{2} a_{b_1}^2, \frac{1}{2} a_{b_1}^2 \right],
\]
\[
\xi_{s_2}' = \left[ -\frac{1}{2} a_{b_1}^2, \frac{1}{2} a_{b_1}^2 \right].
\]  
(30)

Eq. (30) leads to fairly complex relations for a geometrically non-balanced plain-weave, and will not be treated in further detail in the present paper. However, by virtue of the adopted shape functions for the thickness distributions and out-of-plane slice paths, (30) was satisfied for the configurations that will be presented in Section 4.

It should be noted that the adopted shape function for the out-of-plane undulation is identical to that of an infinitely long linear elastic beam with constant flexural rigidity \((EI)\), which is subjected to point forces \((F)\) at equal distances along the centreline, as schematised in Fig. 7. This can be regarded as a simplified representation of the weaving process. Between two subsequent point forces, a linear moment curve will result which leads to the cubic polynomial description of the vertical displacements \((w)\). Obviously, a single fibre bundle slice will not behave exactly like an elastic beam with constant cross-section and flexural rigidity. Furthermore, numerous loading conditions are conceivable during weaving and processing, and interaction between adjacent fibre bundle slices will take place. However, compared to the sinusoidal description most frequently used [9–13,16] – for which no physical motivation is known to the authors – a description using cubic polynomials is much easier to handle, as has been recognised in several recent publications [14,15]. A more detailed schematisation of the forces that are applied to a fabric, and their effect on geometry, has been presented by Olofsson [20]. This approach, however, yields too complex results for the purpose of the present work.

For an extension to other weave types, it is believed that adequate shape functions can be derived using a

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Fig. 7. Schematisation of a linear elastic beam with constant flexural rigidity \((EI)\) subjected to point loads \((F)\) which leads to deflections \((w(x'))\).
consideration similar to the one presented above. This may lead to a significant improvement over existing shape functions applied to, e.g., satin-weave. In literature [9,16,17], a satin-weave is generally modeled as a collection of curved (sinusoidal) segments for the interlacing, which are connected by straight segments for the floats. This approach, which neglects the bending stiffness of the fibre bundle, is not very realistic as can be observed from Fig. 4(b). In deriving the shape functions mentioned above, it should furthermore be noted that contact between two fibre bundles at a crossover region is not a prerequisite. In Fig. 4(b), no contact takes place between the float and the overcrossed yarn located in the middle.

3.2. Simplified configuration

The case investigated most frequently in the literature [10–12] is a geometrically balanced, orthogonal plain-weave without gaps. For a geometrically balanced weave, all fibre bundle dimensions of the warp and fill directions are identical and subscripts w and f can be omitted in the geometric relations. Furthermore, applying (27) to a geometrically balanced plain-weave yields

\[ A_c = \frac{1}{2} t. \]  \hfill (31)

This implies that neither amplitude is required as input parameter for a geometrically balanced plain-weave.

For a plain-weave without gaps between the fibre bundles, the following relation applies:

\[ a_w^* = \frac{1}{2}. \]  \hfill (32)

For an orthogonal plain-weave (θ = 90°), the phase shift between parallel fibre slices disappears and

\[ \zeta_w^* = \zeta_f^*. \]  \hfill (33)

Combination of (28), (29), and (31)–(33) yields

\[ \omega = k_2 \frac{t}{2T} [-48 \zeta_i^* + 96 \text{sign}(\zeta_i) \zeta_i^*], \quad \zeta_i^* \in \left[ \frac{1}{2}, \frac{3}{2} \right]. \]  \hfill (34)

The factor \( k_2 \) is given by (21). For this very straightforward configuration, the crimp angle is independent of \( q' \) and all fibre paths are parallel, which implies constant cross-sections.

4. Examples

4.1. Balanced plain-weave before and after stretching

As a first example, the effects of stretching on a carbon fibre (CF) plain-weave (Ten Cate CD200) are investigated. A 3-D plot of the outer surfaces of the plain-weave is depicted in Fig. 8(a). In the undeformed configuration, the plain-weave is geometrically balanced. Required input parameters are presented in the figure. These have been measured directly on laminate cross-section photomicrographs (Fig. 9(a)), which will be the subject of further investigation below.

The distribution of out-of-plane crimp angles throughout the impregnated warp and fill fibre bundles (yarns) is depicted in Fig. 8(b) and (c), respectively, and is calculated with (29). Every contour in the figures reflects an increment of 0.02 rad for the crimp angle. Notice that, by virtue of the periodicity, the intervals \( \zeta_i^* \in \left[ \frac{1}{2}, \frac{3}{2} \right] \), depicted in the figures, correspond to the intervals \( \zeta_i^* \in \left[ \frac{1}{2}, \frac{3}{2} \right] \), which were considered in the calculations. The maximum out-of-plane crimp angles amount to \( |\omega(\zeta_i^* = \pm \frac{1}{2}, q' = 0)| = 0.113 \) rad at the centreline. Maximum crimp angles of the outer fibres for these cross-sections are only 5% smaller, with \( |\omega(\pm \frac{1}{2}, \pm \frac{1}{2})| = 0.108 \) rad.

The CD200 plain-weave is schematised after a stretching deformation in Fig. 8(d). Warp yarns have been straightened and, consequently, the fill yarns have been pushed upwards and downwards, i.e. undulation becomes more pronounced. The latter deformation mode is denoted as crimping [5]. Furthermore, cross-section dimensions have changed for both yarn directions. Yarn cross-sections of the stretched direction exhibit a rounder shape, while cross-sections of the crimped direction have flattened [5]. As a result of the inflicted deformations, the weave is not anymore geometrically balanced. This can be concluded from the geometric input parameters, which were measured from Fig. 9(b) and (c).

The straightened warp yarn exhibits very little crimp (Fig. 8(e)). In this case, the maximum out-of-plane crimp angles are observed for the outer fibres and merely amount to \( |\omega(\pm \frac{1}{2}, 0)| = 0.033 \) rad. Significant variations in crimp between the individual fibre slices of the fill yarn have resulted from crimping, as can be observed from Fig. 8(f). Maximum crimp is observed at the centreline, with \( |\omega(\pm \frac{1}{2}, 0)| = 0.202 \) rad. Outer fibres of the same cross-sections exhibit a maximum crimp angle which is 38% smaller, with \( |\omega(\pm \frac{1}{2}, \pm \frac{1}{2})| = 0.124 \) rad.

A comparison between the 3-D geometry model and laminate cross-section photomicrographs is presented in Fig. 9. For the model, outer contours are depicted for all yarns, and midplanes are depicted for the crossed yarns. The laminate is built up from six identically orientated CD200 reinforcement layers, which are impregnated and surrounded with a poly-phenylene sulfide (PPS) thermoplastic matrix. A detail of a cross-section along the yarn centreline, which was cut out from the undeformed laminate, is depicted in Fig. 9(a). Good correspondence is observed with the adopted shape functions for the undulation and the thickness distribution. Cross-sections cut from a stretched lami-
nate are depicted in Fig. 9(b) and (c). The stretching deformation was carried out by melting the thermoplastic matrix of a laminate specimen in a heat chamber, and subsequently applying a tensile load, as described in [5]. After imposing the stretching deformation, the laminate was not re-consolidated. Consequently, the individual fabric layers are surrounded by gaps. Therefore, only a single impregnated fabric reinforcement layer is depicted in the cross-section photomicrographs. Agreement between theory and 3-D geometry model is good along the mean direction of the crimped yarn (Fig. 9(b)). In Fig. 9(c), full contact is observed between the straightened centreline slice and the undercrossed fill yarn on the right side as well, as has been assumed in Section 3. This cross-section exhibits significant curvature, which is the cause for the variations between individual fibre paths that were predicted with the 3-D model (see Fig. 8(f)). However, the cross-section of the overcrossed fill yarn on the left side has only partly deformed during the applied stretching deformation, and will exhibit less variation between individual fibre paths than predicted.

4.2. Balanced plain-weave before and after a shear deformation

A second example is a CF plain-weave (Ten Cate CD206), for which the geometry is modeled before ($\theta = 90^\circ$) and after an applied shear deformation ($\theta = 54^\circ$). For both configurations, the plain-weave is geometrically balanced. The imposed shear deformation is close to the locking angle of the fabric, which is defined in, among others, [6]. 3-D geometry of the undeformed fabric (Fig. 10(a)) is nearly identical to that of the CD200 plain-weave. The CD206 fabric, however, exhibits larger gaps between the yarns. This is reflected by the yarn width to wavelength ratio, which amounts to $a/l = 0.483$ for the CD200 fabric, and is significantly lower with $a/l = 0.415$ for the CD206 fabric. As observed from the out-of-plane crimp angle distribution.
undeformed, geometrically balanced CF–CD200/PPS laminate

\[ \eta^* = 0 \]

(a)

CF–CD200/PPS laminate, stretched in warp direction

\[ \eta^*_w = 0 \]

(b)

(c)

Fig. 9. Comparison between laminate cross-section photomicrographs and 3-D geometry model for the CD200 carbon plain-weave reinforced laminate: (a) cross-section along a yarn centreline of the undeformed configuration, (b) stretched configuration, cross-section along the centreline of a crimped yarn, (c) stretched configuration, cross-section along the centreline of a straightened yarn.

for the warp yarn (Fig. 10(b)) and the fill yarn (Fig. 10(c)), this difference in geometry leads to slightly higher variations between the individual fibre paths for the CD206 plain-weave. The maximum out-of-plane crimp angles of the centreline slices amount to \(|\omega(\pm\frac{1}{2},0)| = 0.113\) rad. The outer fibres exhibit maximum crimp angles at the same cross-section locations, which are 25% lower with \(|\omega(\pm\frac{1}{2}, \pm\frac{1}{2})| = 0.085\) rad.

The sheared fabric, for which a plot of the outer surfaces is depicted in Fig. 10(d), exhibits slightly higher crimp. This is caused by a decreased distance between parallel yarns, which leads to a lower yarn width and, for constant volume, a higher yarn thickness [5]. Variations between crimp angles for the individual cross-sections are significant, as observed from Fig. 10(e) and (f) for the first warp and fill yarns, respectively. The cause for this is the phase shift of the fibre slices with respect to the centrelines. Maximum values for the out-of-plane crimp angles are observed at the centrelines and amount to \(|\omega(\pm\frac{1}{2},0)| = 0.132\) rad. Outer fibres for these particular yarn cross-sections exhibit crimp angles which are 43% lower, with \(|\omega(\pm\frac{1}{2}, \pm\frac{1}{2})| = 0.075\) rad. The difference between centreline slices and outer fibres is even more pronounced for the warp and fill yarn cross-sections at the crossover points. Outer fibres exhibit crimp angles that amount to \(|\omega(0, \pm\frac{1}{2})| = 0.073\) rad. The sign of the crimp angles is opposite for the outer fibres at \(\eta^* = -\frac{1}{4}\) and \(\eta^* = \frac{1}{2}\). The centreline slice exhibits no inclination at this cross-section location. Differences between the maximum crimp angles of the individual slices are comparable to those observed for the undeformed configuration.

A comparison with details from laminate cross-section photomicrographs is presented in Fig. 11. Both the undeformed and the sheared laminates contain 6 CD206 reinforcement layers with identical orientations. The latter configuration was obtained by melting the surrounding PPS thermoplastic matrix in an oven and applying a nearly pure shear deformation with a hinged four-bar frame [6]. Subsequently, the laminate was re-consolidated. Details of laminate cross-sections along the mean yarn direction are depicted in Fig. 11(a), for the undeformed configuration, and Fig. 11(b), for the sheared configuration. Agree-
Fig. 10. 3-D geometry model for a balanced carbon plain-weave (Ten Cate CD206) before and after shearing: (a) 3-D plot of the undeformed plain-weave with input parameters, (b) out-of-plane crimp angle distribution for the first warp fibre bundle and (c) the first fill fibre bundle, (d) 3-D plot of the sheared plain-weave with input parameters, (e) out-of-plane crimp angle distribution for the first warp fibre bundle and (f) the first fill fibre bundle.

ment with the proposed shape functions is good, and full contact between the yarns is observed along the centreline, as has been assumed for the geometry model. Interesting observations can be made for the laminate cross-section normal to the mean yarn direction which is depicted in Fig. 11(c). It is observed that the cross-section midplanes exhibit both a significant curvature and twist, as can be concluded from Fig. 10(e) and (f) as well. These deformed configurations are very accurately predicted with the 3-D model.

4.3. Non-balanced plain-weave before and after a shear deformation

As a final example, a non-balanced glass plain-weave is considered in undeformed ($\theta = 90^\circ$) and sheared ($\theta = 61^\circ$) states. A 3-D model of the plain-weave, as present in the undeformed laminate, is shown in Fig. 12(a). Out-of-plane crimp angle distributions are shown for the first warp yarn in Fig. 12(b), and for the first fill yarn in Fig. 12(c). The maximum crimp angle inside the warp yarn amounts to $|\omega_w(\pm \frac{1}{2}, 0)| = 0.107$ rad along the centreline. Maximum crimp angles for the outer fibres are 34% smaller at these cross-section locations, with $|\omega_w(\pm \frac{1}{2}, \pm \frac{1}{2})| = 0.070$ rad. The fill yarns exhibit higher crimp than the warp yarns, with maximum values for the crimp angle of $|\omega_f(\pm \frac{1}{2}, 0)| = 0.168$ rad. For the outer fibres, the maximum crimp angles observed at the identical cross-section locations are 17% smaller, with $|\omega_f(\pm \frac{1}{2}, \pm \frac{1}{2})| = 0.140$ rad.

The sheared plain-weave, depicted in Fig. 12(d), exhibits slightly higher crimp in warp and fill directions than the undeformed configuration, as was elucidated for the CD206 plain-weave before. For the warp yarn (Fig. 12(e)), the maximum crimp angles, observed along
undeformed, geometrically balanced CF–CD206/PPS laminate

\[ \eta' = 0 \]

(a)

sheared, geometrically balanced CF–CD206/PPS laminate: \( \theta = 54^\circ \)

\[ \eta' = 0 \]

(b)

\[ \xi' = 0 \]

(c)

Fig. 11. Comparison between laminate cross-section photomicrographs and 3-D geometry model for the CD206 carbon plain-weave reinforced laminate: (a) cross-section along a yarn centreline of the undeformed configuration, (b) cross-section along a yarn centreline of the sheared configuration, (c) cross-section perpendicular to the mean yarn direction of the sheared configuration.

the centreline, amount to \( |\omega_y(\pm\frac{1}{2}, 0)| = 0.126 \) rad, which is 21% higher than for the outer fibres at this location, with \( |\omega_y(\pm\frac{1}{2}, \pm\frac{1}{2})| = 0.100 \) rad. For the fill yarn, this variation amounts to 38%, with maximum crimp angles \( |\omega_f(\pm\frac{1}{2}, 0)| = 0.176 \) rad, for the centreline, and \( |\omega_f(\pm\frac{1}{2}, \pm\frac{1}{2})| = 0.108 \) rad, for the outer fibres at the same cross-section locations. Again, the phase shift of outer fibre slices with respect to the centreline accounts for these significant variations in fibre orientation throughout an individual cross-section. Most significantly, the fill yarn cross-sections at the crossover points exhibit outer fibres with inclination \( |\omega_f(0, \pm\frac{1}{2})| = 0.098 \) rad. This orientation is, as noted before, opposite in sign for the two outer fibre locations, while the out-of-plane crimp angle of the centreline slice equals zero.

A comparison with details from laminate cross-section photomicrographs is presented in Fig. 13. The laminates consist of PolyAmid 6 (PA6) thermoplastic matrix reinforced with five glass plain-weaves with identical orientations. The shear deformation was applied as described for the CD206 plain-weave [6]. Good correspondence between 3-D model and experiment is observed for all laminate cross-sections. Laminate cross-sections normal to the mean yarn directions are not depicted in Fig. 13, as contrast between the glass fibre cross-sections and surrounding PA6 matrix is difficult to reproduce.

5. Discussion and outlook

A 3-D geometry model has been presented for use as an interface between draping simulations on the one hand, and calculation of effective material properties and macro-mechanical analysis of fabric reinforced products on the other hand. Draping the fabric over the product mould leads to significant fibre bundle reorientations and cross-section deformations, which can be described with the proposed model. Every fibre bundle was described in its local coordinates, which are placed along the mean fibre bundle direction, by: (i) the in-plane centreline path, (ii) the double curved horizontal midplane and (iii) the cross-section thickness distribution. Angles between the local and mean fibre bundle directions were assumed to be moderate. The relative location of a fibre inside a bundle was assumed constant. Furthermore, fibre packing was assumed to be constant for every cross-section. With these assumptions, it was
shown that the fibre distribution and individual fibre paths can easily be derived with the modeling scheme, as a function of geometric input parameters and adopted shape functions for the undulation and thickness distribution. The fibre distribution and orientations are required for calculation of effective material properties.

The modeling scheme was applied to geometrically non-balanced plain-weaves. Adequate shape functions for the thickness distribution and undulation path were introduced, based on experimental observations and a simplified consideration of the weaving process. Required geometric input parameters were measured directly from cross-section photomicrographs, which were presented for plain-weave reinforced laminates in undeformed, sheared and stretched configurations. A comparison was made between the 3-D models and the actual laminate reinforcements. Good correspondence was observed between theory and experiment. Very significant variations in fibre orientation throughout individual cross-sections were found. For the stretched carbon plain-weave considered in Section 4.1, differences in out-of-plane crimp angles between the individual fibres were predicted of up to 38%. For the sheared, non-balanced glass plain-weave reinforcement considered in Section 4.3, a difference in out-of-plane crimp angles of 0.196 rad was predicted between the outer fibres of an individual cross-section. The crimp angles for the outer fibres were identical but opposite in sign. The maximum crimp angle, observed at a different centreline location, merely amounted to 0.176 rad. The variations described in Section 4 cannot be modeled with conventional modeling schemes, which assume all fibre paths to be parallel to the centreline path of the fibre bundle. Furthermore, assuming constant cross-sections, it will be very difficult to describe sheared plain-weaves realistically without leading to cross-section violations between the interlaced fibre bundles.

The deformed fibre bundle configurations treated above were defined as a combination of three modes,
Fig. 13. Comparison between laminate cross-section photomicrographs and 3-D geometry model for the glass plain-weave reinforced laminate (Bayer Polystal): (a) cross-section along the fill yarn centreline of the undeformed configuration, (b) cross-section along the warp yarn centreline of the undeformed configuration, (c) cross-section along the fill yarn centreline of the sheared configuration, (d) cross-section along the warp yarn centreline of the sheared configuration.

namely: (i) out-of-plane undulation, (ii) midplane curvature and (iii) twist of the midplane. More complex weave types, such as satins, may additionally exhibit in-plane undulation and variations in cross-section dimensions, which may occur for either constant or variable fibre packing ratios. It is expected that these weave types and their deformed configurations can be described with the present modeling scheme as well. 3-D textiles with fibre bundle paths that locally deviate strongly from the mean direction, however, cannot be modeled with the proposed architecture. For such textiles, the assumption of moderate deviations in fibre orientation from the mean fibre bundle direction needs to be reconsidered.

A link between a drape simulation, the 3-D geometry model and micromechanics calculations of the effective stiffness properties is the focus of present work. In contrast to describing the deterministic fibre bundle reorientations and deformations inflicted by draping the fabric, a different application of the 3-D model may be to predict the effects of unwanted deformations. Such an application may eventually be translated into useful bounds on the material properties that are likely to be obtained inside a structure. Many (unwanted) geometry deviations inside a fabric have been described by Pastore [21]. A further possibility would be to incorporate the 3-D model directly into the draping simulation. Initial work in this direction has been carried out by Robitaille et al. [22].
Acknowledgements

Ten Cate Advanced Composites, the Nederlandse Organisatie voor Wetenschappelijk Onderzoek (NWO) and the Max Planck Gesellschaft are gratefully acknowledged for their contributions to the present work.

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